Important Equations

General Functions

$$\frac{de^{at}}{dt} = ae^{at} \qquad \int e^{at}dt = \frac{1}{a}e^{at}$$

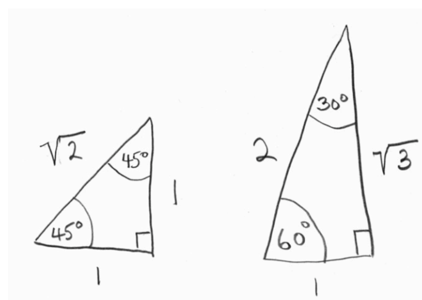
$$e^{j\omega t} = \cos\omega t + j\sin\omega t \quad \text{(Euler's Identity)}$$

$$\cos\omega t = \mathrm{Re}\{e^{j\omega t}\} = \frac{e^{j\omega t} + e^{-j\omega t}}{2} \qquad \sin\omega t = \mathrm{Im}\{e^{j\omega t}\} = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$Ev\{x(t)\} = \frac{x(t) + x(-t)}{2} \qquad Od\{x(t)\} = \frac{x(t) - x(-t)}{2}$$

$$x(t) = Ev\{x(t)\} + Od\{x(t)\}$$

$$\partial(t) = \begin{cases} \infty, & \text{t} = 0 \\ 0, & \text{t} \neq 0 \end{cases} \qquad \int_{-\infty}^{+\infty} \partial(t) = 1 \qquad \partial[n] = \begin{cases} 1, & \text{n} = 0 \\ 0, & \text{n} \neq 0 \end{cases}$$



$$u(t) = \begin{cases} 1, & \text{t} > 0 \\ 0, & \text{t} < 0 \\ \text{indeterminate}, & \text{t} = 0 \end{cases} \qquad u[n] = \begin{cases} 1, & \text{t} \geq 0 \\ 0, & \text{t} < 0 \end{cases}$$

$$x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau \qquad x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k] \quad \text{(Convolution)}$$

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt \qquad E = \sum_{n=-\infty}^{+\infty} |x[n]|^2 \quad \text{(Energy)}$$

$$x(t) = x(t - T_0) \qquad \omega_0 = \frac{2\pi}{T_0} \qquad x[n] = x[n - N_0] \quad \text{(Periodic)}$$

$$P = \frac{1}{T_0}\int_{T_0} |x(t)|^2 dt \qquad P = \frac{1}{N_0}\sum_{n=\langle N_0 \rangle} |x[n]|^2 \quad \text{(Power)}$$

Fourier Series

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \qquad a_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt$$

$$x(t) \xleftrightarrow{Fs} a_k \qquad y(t) \xleftrightarrow{Fs} b_k \qquad a_k = a_{-k}^{*} \quad \text{for real } x(t)$$

$$Ax(t) + By(t) \xleftrightarrow{Fs} Aa_k + Bb_k \qquad x(-t) \xleftrightarrow{Fs} a_{-k}$$

$$x(\alpha t) \overset{Fs}{\longleftrightarrow} a_k, \quad \alpha > 0$$

$$\frac{dx(t)}{dt} \overset{Fs}{\longleftrightarrow} jk\omega_0 a_k \qquad\qquad \int x(t)dt \overset{Fs}{\longleftrightarrow} \frac{1}{jk\omega_0} a_k$$

$$x(t-\tau) \overset{Fs}{\longleftrightarrow} e^{-jk\omega_0\tau} a_k$$

$$x(t)y(t) \overset{Fs}{\longleftrightarrow} \sum_{n=-\infty}^{+\infty} a_n b_{k-n}$$

$$\frac{1}{T_0}\int_{T_0} |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2 \quad \text{(Parseval's Relation)}$$

$$x(t) = 1 \overset{Fs}{\longleftrightarrow} a_0 = 1 \qquad\qquad x(t) = \sum_{n=-\infty}^{+\infty} \partial(t - nT_0) \overset{Fs}{\longleftrightarrow} a_k = \frac{1}{T_0}$$

$$x(t) = \cos(k\omega_0 t) \overset{Fs}{\longleftrightarrow} a_k = \frac{1}{2},\ a_{-k} = \frac{1}{2} \qquad\qquad x(t) = \sin(k\omega_0 t) \overset{Fs}{\longleftrightarrow} a_k = -\frac{j}{2},\ a_{-k} = \frac{j}{2}$$

Fourier Transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega) e^{j\omega t} d\omega \qquad\qquad X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$x(t) \overset{F}{\longleftrightarrow} X(\omega) \qquad\qquad X(\omega) = X^*(-\omega) \quad \text{for real } x(t)$$

$$Ax(t) + By(t) \overset{F}{\longleftrightarrow} AX(\omega) + BY(\omega) \qquad\qquad x(-t) \overset{F}{\longleftrightarrow} X(-\omega)$$

$$x(\alpha t) \overset{F}{\longleftrightarrow} \frac{1}{\alpha} X\left(\frac{\omega}{\alpha}\right) \qquad \alpha > 0$$

$$\frac{dx(t)}{dt} \overset{F}{\longleftrightarrow} j\omega X(\omega) \qquad\qquad \int x(t)dt \overset{F}{\longleftrightarrow} \frac{1}{j\omega} X(\omega)$$

$$x(t-\tau) \overset{F}{\longleftrightarrow} e^{-j\omega\tau} X(\omega)$$

$$x(t)y(t) \overset{F}{\longleftrightarrow} \frac{1}{2\pi} X(\omega) * Y(\omega) \qquad\qquad x(t) * y(t) \overset{F}{\longleftrightarrow} X(\omega)Y(\omega)$$

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(\omega)|^2 d\omega \quad \text{(Parseval's Relation)}$$

$$X(\omega) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

(relation of Fourier Transform to Fourier Series)

$$x(t) = 1 \xleftrightarrow{F} X(\omega) = 2\pi\partial(\omega) \qquad x(t) = \partial(t) \xleftrightarrow{F} X(\omega) = 1$$

$$x(t) = \cos(\omega_1 t) \xleftrightarrow{F} X(\omega) = \pi\partial(\omega - \omega_1) + \pi\partial(\omega + \omega_1)$$

$$x(t) = \sin(\omega_1 t) \xleftrightarrow{F} X(\omega) = -j\pi\partial(\omega - \omega_1) + j\pi\partial(\omega + \omega_1)$$

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0 & |t| > T_1 \end{cases} \xleftrightarrow{F} X(\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

$$\text{sinc}\lambda = \frac{\sin\pi\lambda}{\pi\lambda}$$

Complex Impedance

$$Z_c = \frac{1}{j\omega C} \qquad Z_L = j\omega L \qquad Z_R = R$$

$$Z_S = Z_1 + Z_2 \text{ (series)} \qquad Z_P = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} \text{ (parallel)}$$