Important Equations

General Functions

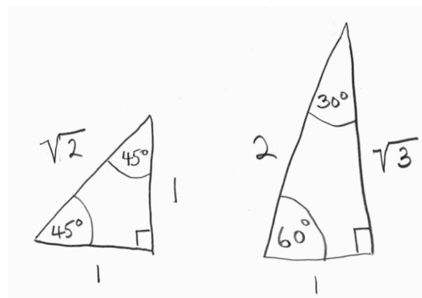
$$\frac{de^{at}}{dt} = ae^{at} \qquad \int e^{at}dt = \frac{1}{a}e^{at}$$

$$e^{j\omega t} = \cos\omega t + j\sin\omega t \qquad \text{(Euler's Identity)}$$

$$\cos\omega t = \mathrm{Re}\left\{ e^{j\omega t}\right\} = \frac{e^{j\omega t} + e^{-j\omega t}}{2} \qquad \sin\omega t = \mathrm{Im}\left\{ e^{j\omega t}\right\} = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$Ev\{x(t)\} = \frac{x(t) + x(-t)}{2} \qquad Od\{x(t)\} = \frac{x(t) - x(-t)}{2}$$

$$x(t) = Ev\{x(t)\} + Od\{x(t)\}$$



$$\partial(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases} \qquad \int_{-\infty}^{+\infty} \partial(t) = 1 \qquad \partial[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \\ \text{indeterminate}, & t = 0 \end{cases} \qquad u[n] = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau \qquad x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k] \qquad \text{(Convolution)}$$

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt \qquad E = \sum_{n=-\infty}^{+\infty} |x[n]|^2 \qquad \text{(Energy)}$$

$$x(t) = x(t - T_0) \qquad \omega_0 = \frac{2\pi}{T_0} \qquad x[n] = x[n - N_0] \qquad \text{(Periodic)}$$

$$P = \frac{1}{T_0}\int_{T_0} |x(t)|^2 dt \qquad P = \frac{1}{N_0}\sum_{n=\langle N_0\rangle} |x[n]|^2 \qquad \text{(Power)}$$

Fourier Series

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \qquad a_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt$$

$$x(t) \overset{Fs}{\longleftrightarrow} a_k \qquad y(t) \overset{Fs}{\longleftrightarrow} b_k \qquad a_k = a_{-k}^{*} \quad \text{for real } x(t)$$

$$Ax(t) + By(t) \overset{Fs}{\longleftrightarrow} Aa_k + Bb_k \qquad x(-t) \overset{Fs}{\longleftrightarrow} a_{-k}$$

$$x(\alpha t) \overset{Fs}{\longleftrightarrow} a_k, \quad \alpha > 0$$

$$\frac{dx(t)}{dt} \overset{Fs}{\longleftrightarrow} jk\omega_0 a_k \qquad \int x(t)dt \overset{Fs}{\longleftrightarrow} \frac{1}{jk\omega_0} a_k$$

$$x(t-\tau) \overset{Fs}{\longleftrightarrow} e^{-jk\omega_0\tau} a_k$$

$$x(t)y(t) \overset{Fs}{\longleftrightarrow} \sum_{n=-\infty}^{+\infty} a_n b_{k-n}$$

$$\frac{1}{T_0}\int_{T_0} |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2 \quad \text{(Parseval's Relation)}$$

$$x(t) = 1 \overset{Fs}{\longleftrightarrow} a_0 = 1 \qquad x(t) = \sum_{n=-\infty}^{+\infty} \partial(t - nT_0) \overset{Fs}{\longleftrightarrow} a_k = \frac{1}{T_0}$$

$$x(t) = \cos(k\omega_0 t) \overset{Fs}{\longleftrightarrow} a_k = \frac{1}{2},\ a_{-k} = \frac{1}{2} \qquad x(t) = \sin(k\omega_0 t) \overset{Fs}{\longleftrightarrow} a_k = -\frac{j}{2},\ a_{-k} = \frac{j}{2}$$

Fourier Transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega) e^{j\omega t} d\omega \qquad X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$x(t) \overset{F}{\longleftrightarrow} X(\omega) \qquad X(\omega) = X^*(-\omega) \quad \text{for real } x(t)$$

$$Ax(t) + By(t) \overset{F}{\longleftrightarrow} AX(\omega) + BY(\omega) \qquad x(-t) \overset{F}{\longleftrightarrow} X(-\omega)$$

$$x(\alpha t) \overset{F}{\longleftrightarrow} \frac{1}{\alpha} X\left(\frac{\omega}{\alpha}\right) \qquad \alpha > 0$$

$$\frac{dx(t)}{dt} \overset{F}{\longleftrightarrow} j\omega X(\omega) \qquad \int x(t)dt \overset{F}{\longleftrightarrow} \frac{1}{j\omega} X(\omega)$$

$$x(t-\tau) \overset{F}{\longleftrightarrow} e^{-j\omega\tau} X(\omega)$$

$$x(t)y(t) \overset{F}{\longleftrightarrow} \frac{1}{2\pi} X(\omega) * Y(\omega) \qquad x(t) * y(t) \overset{F}{\longleftrightarrow} X(\omega)Y(\omega)$$

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(\omega)|^2 d\omega \quad \text{(Parseval's Relation)}$$

$$X(\omega) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

(relation of Fourier Transform to Fourier Series)

$$x(t) = 1 \overset{F}{\longleftrightarrow} X(\omega) = 2\pi\partial(\omega) \qquad x(t) = \partial(t) \overset{F}{\longleftrightarrow} X(\omega) = 1$$

$$x(t) = \cos(\omega_1 t) \overset{F}{\longleftrightarrow} X(\omega) = \pi\partial(\omega - \omega_1) + \pi\partial(\omega + \omega_1)$$

$$x(t) = \sin(\omega_1 t) \overset{F}{\longleftrightarrow} X(\omega) = -j\pi\partial(\omega - \omega_1) + j\pi\partial(\omega + \omega_1)$$

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0 & |t| > T_1 \end{cases} \overset{F}{\longleftrightarrow} X(\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

$$\operatorname{sinc}\lambda = \frac{\sin \pi\lambda}{\pi\lambda}$$

Complex Impedance

$$Z_c = \frac{1}{j\omega C} \qquad Z_L = j\omega L \qquad Z_R = R$$

$$Z_S = Z_1 + Z_2 \text{ (series)} \qquad Z_P = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} = \frac{Z_1 Z_2}{Z_1 + Z_2} \text{ (parallel)}$$

Laplace Transform

$s = \sigma + j\omega$

$X(s) = \int_{-\infty}^{+\infty} x(t) e^{-st} dt$

$x(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s) e^{st} ds$

$Ax_1(t) + Bx_2(t) \overset{L}{\longleftrightarrow} \mathrm{A}X_1(s) + BX_2(s)$

$x(t - t_1) \overset{L}{\longleftrightarrow} \mathrm{e}^{-\mathrm{s}\mathrm{t}_1} X(s)$

$x(t) * h(t) \overset{L}{\longleftrightarrow} X(s) Y(s)$

$x(at) \overset{L}{\longleftrightarrow} \frac{1}{\mathrm{a}} X\left(\frac{s}{a}\right), \quad a > 0$

$\frac{dx(t)}{dt} \overset{L}{\longleftrightarrow} sX(s)$

$\int_{-\infty}^{t} x(t) dt \overset{L}{\longleftrightarrow} \frac{1}{s} X(s)$

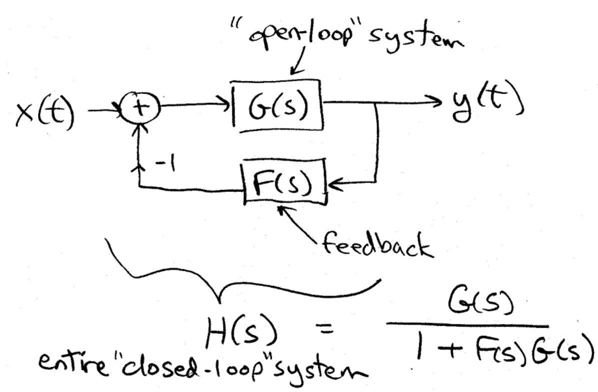
$x(t) = e^{-at} u(t) \overset{L}{\longleftrightarrow} X(s) = \frac{1}{s + a}$

$x(t) = u(t) \overset{L}{\longleftrightarrow} X(s) = \frac{1}{s}$

$x(t) = \partial(t) \overset{L}{\longleftrightarrow} X(s) = 1$

$x(t) = \partial(t - t_1) \overset{L}{\longleftrightarrow} X(s) = e^{-st_1}$

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k} \quad \Rightarrow \quad H(s) = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k}$$



Partial Fractions

$$X(s) = \frac{c_1}{(s + a)} + \frac{c_2}{(s + b)} = \frac{c_1(s + b) + c_2(s + a)}{(s + a)(s + b)}$$

$$c_1 = \left[X(s)(s + a)\right]_{s=-a} = \left[\frac{c_1(s + b) + c_2(s + a)}{(s + b)}\right]_{s=-a}$$

Geometric Series

Infinite:

for $|\alpha| < 1, \quad \sum_{n=0}^{\infty} \alpha^n = \frac{1}{1 - \alpha}$

Finite:

for any $\alpha, \quad \sum_{n=0}^{N} \alpha^n = \frac{1 - \alpha^{N+1}}{1 - \alpha}$

Discrete Time Fourier Series

$x[n] = x[n+N]$ assume $\omega_s = N\omega_0 = 2\pi$ $a_k = a_{k+N}$

$$x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\omega_0 t} \qquad a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]\, e^{-jk\omega_0 t}$$

$$w = e^{j\frac{2\pi}{N}} \qquad w^N = 1$$

$$F = \begin{bmatrix} w^0 & w^0 & w^0 & . & . \\ w^0 & w^1 & w^2 & . & . \\ w^0 & w^2 & w^4 & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix} \qquad F^{-1} = \frac{1}{N}\begin{bmatrix} w^0 & w^0 & w^0 & . & . \\ w^0 & w^{-1} & w^{-2} & . & . \\ w^0 & w^{-2} & w^{-4} & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix}$$

Discrete Time Fourier Transform

assume $\omega_s = 2\pi$ $X(\omega) = X(\omega + 2\pi)$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(\omega) e^{j\omega n} d\omega \qquad X(\omega) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\omega n}$$

$$x[n-n_0] \overset{F}{\longleftrightarrow} e^{-j\omega n_0} X(\omega)$$

Sampling and Aliasing

For $x(t)$ sampled at ω_s $\omega_{\max} < \frac{\omega_s}{2}$ (Nyquist frequency) to avoid aliasing.

Z Transform

$$z = re^{j\omega} \qquad X(z) = \sum_{n=-\infty}^{+\infty} x[n]\, z^{-n} \qquad x[n] = \frac{1}{2\pi j}\oint X(z) z^{n-1} dz$$

$$\partial[n] \overset{z}{\longleftrightarrow} 1 \qquad x[n-n_0] \overset{z}{\longleftrightarrow} z^{-n_0} X(z) \qquad a^n u[n] \overset{z}{\longleftrightarrow} \frac{z}{z-a}$$