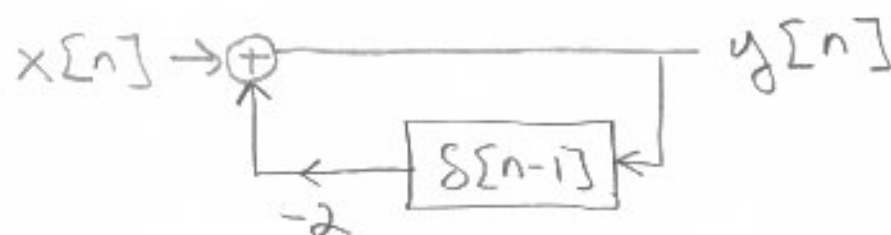


Question

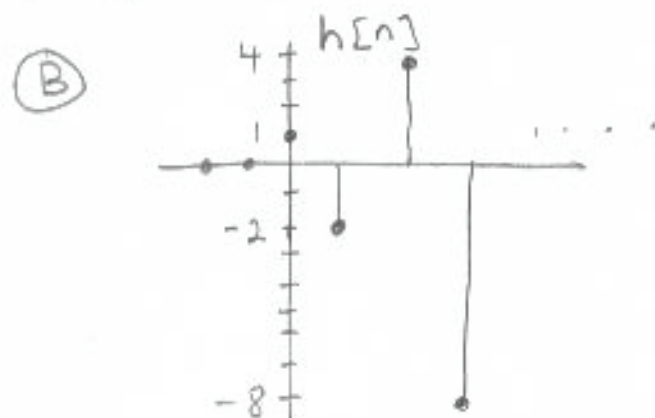
For the system



- (A) write the system equation.
- (B) sketch the impulse response
- (C) is it IIR or FIR?

Answer

(A) $y[n] = x[n] - 2y[n-1]$



(C) IIR

Question

The input to a LTI system is

$$x(t) = \cos(3t) - \sin(3t) + \cos(9t)$$

(A) what is the fundamental freq. ω_0 ?

(B) what is the fundamental period T_0 ?

Which of the following are possible outputs?

(C) $y(t) = 6 + \cos(9t)$

(D) $y(t) = \cos(3t) + 2\cos(9t)$

(E) $y(t) = 3\cos(3t - 1)$

(F) $y(t) = 2\sin^2(3t)$

Answer

(A) $\omega_0 = 3$

(B) $T_0 = \frac{2\pi}{3}$

(D) and (E) are possible

(C) and (F) are not.

Question

Find the fourier transform $X(\omega)$
for
 $x(t) = 1 + e^{-t}u(t)$

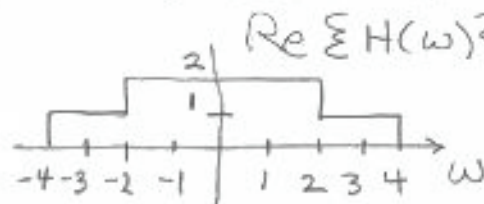
Answer

$$\begin{aligned} 1 &\xleftrightarrow{F} 2\pi\delta(\omega) \\ e^{-t}u(t) &\xleftrightarrow{F} \int_{-\infty}^{+\infty} e^{-t}u(t)e^{-j\omega t}dt = \\ &\int_0^{\infty} e^{-(1+j\omega)t}dt = -\frac{1}{1+j\omega} e^{-(1+j\omega)t} \Big|_0^{\infty} = \\ &\frac{1}{1+j\omega} \end{aligned}$$

$$X(\omega) = 2\pi\delta(\omega) + \frac{1}{1+j\omega}$$

Question

A system has a frequency
response



with the imaginary part $\text{Im}\{H(w)\} = 0$

Given an input $x(t) = 2 - \sin t + \cos 3t$

- Ⓐ what is the Fourier Transform, $X(w)$?
Ⓑ what is the output signal, $y(t)$?
Ⓒ what is its Fourier Transform, $Y(w)$?
(hint: $Y(w) = X(w)H(w)$)

Answer

Ⓐ $X(w) = 4\pi\delta(w) + \pi\delta(1-w) - \pi\delta(1+w)$
 $+ \pi\delta(3-w) - \pi\delta(3+w)$

Ⓑ $4 - 2\sin t + \cos 3t$

Ⓒ $8\pi\delta(w) + 2\pi\delta(1-w) - 2\pi\delta(1+w)$
 $+ \pi\delta(3-w) - \pi\delta(3+w)$

Question

The impulse response of a system is

$$h(t) = u(t)$$

(A) Find its Fourier Transform $H(\omega)$.

(B) Since $x(t) * h(t) = y(t) \Rightarrow X(\omega)H(\omega) = Y(\omega)$

What does this system do?

- 1) Delay
- 2) Integrate
- 3) Differentiate
- 4) Amplify

Answer

$$\begin{aligned} \text{(A)} \quad H(\omega) &= \int_{-\infty}^{+\infty} h(t) e^{-j\omega t} dt = \int_{-\infty}^{+\infty} u(t) e^{-j\omega t} dt \\ &= \int_0^{+\infty} e^{-j\omega t} dt = -\frac{1}{j\omega} e^{-j\omega t} \Big|_0^{\infty} = \frac{1}{j\omega} \end{aligned}$$

(B) integrate

Question

Compute the Fourier series of

$$x(t) = -3 + \cos(\pi t) - 2\sin(2\pi t)$$

what is ω_0 ?

what is the average power?
(use Parseval's relation)

Answer

$$a_0 = -3$$

$$a_1 = \frac{1}{2} \quad a_{-1} = \frac{1}{2}$$

$$a_2 = j \quad a_{-2} = -j$$

$$\omega_0 = \pi$$

$$P = \sum_{k=-\infty}^{\infty} |a_k|^2 = |-3|^2 + \left|\frac{1}{2}\right|^2 + \left|\frac{1}{2}\right|^2 + |j|^2 + |-j|^2$$

$$= 9 + \frac{1}{4} + \frac{1}{4} + 1 + 1$$

$$= 11\frac{1}{2}$$

Question

The Fourier series of $x(t)$ is

$$a_0 = 6$$

$$a_1 = 1 + \frac{j}{2} \quad a_{-1} = 1 - \frac{j}{2}$$

$$a_2 = -3 \quad a_{-2} = -3$$

(A) what is $x(t)$, if $\omega_0 = 4$

(B) what is the Fourier Series of $\frac{dx(t)}{dt}$?

(C) what is the Fourier Series of $2x(t - \frac{\pi}{2})$?

Answer

(A) $x(t) = 6 + 2\cos(4t) - \sin(4t) - 6\cos(8t)$

(B) $\frac{dx(t)}{dt} \xleftrightarrow{Fs} jk\omega_0 a_k$

$$a_0 = 0$$

$$a_1 = 4j - 2 \quad a_{-1} = -4j - 2$$

$$a_2 = -24j \quad a_{-2} = 24j$$

(C) $x(t - \frac{\pi}{2}) \xleftrightarrow{Fs} e^{-jk\omega_0 \frac{\pi}{2}} a_k$

$$a_0 = 12$$

$$a_1 = 2 + j \quad a_{-1} = 2 - j$$

$$a_2 = -3 \quad a_{-2} = -3$$

Question

Given $x(t) = \sin(t/2\pi)$

$$y(t) = \cos(t/2\pi)$$

- (A) what is the fundamental freq. ω_0
- (B) what is the fundamental period T_0
- (C) find Fourier Series of $x(t)$ and $y(t)$
- (D) what is the average power of each signal (use Fourier Series)
- (E) should shifting a signal change its average power?

Answer

(A) $\omega_0 = 1/2\pi$

(B) $T_0 = 1$

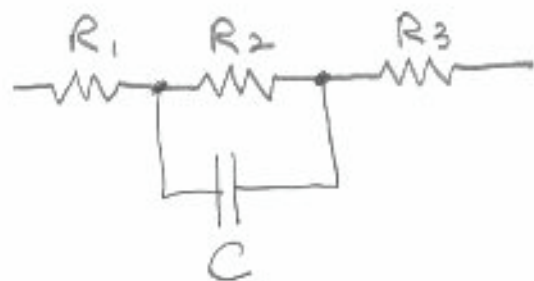
(C) $x(t) \xleftrightarrow{Fs} a_1 = -j/2$ $a_{-1} = +j/2$	(D) power of $x(t) =$ $(-j/2)^2 + (j/2)^2 = \frac{1}{2}$
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$y(t) \xleftrightarrow{Fs} b_1 = 1/2$ $b_{-1} = 1/2$	power of $y(t) =$ $(1/2)^2 + (1/2)^2 = \frac{1}{2}$
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(E) shifting should not change it.

Question

① What is the impedance of this circuit?



② what is the impedance when $\omega = 0$?

③ what is the impedance when $\omega = \infty$?

④ explain ② and ③ in English.

Answer

$$\textcircled{A} Z = R_1 + \frac{1}{\frac{1}{R_2} + j\omega C} + R_3 = R_1 + \frac{R_2}{1 + j\omega R_2 C} + R_3$$

$$\textcircled{B} Z|_{\omega=0} = R_1 + R_2 + R_3$$

$$\textcircled{C} Z|_{\omega=\infty} = R_1 + R_3$$

④ At $\omega = 0$, the cap has ∞ impedance and you have 3 resistors in series.
At $\omega = \infty$ The cap shorts out R_2 leaving just R_1 and R_3 in series.