

Question

$$x(t) = 2e^{2t}$$

$$y(t) = \delta(t+2)$$

$$x(t) * y(t) = ?$$

Answer

$$\begin{aligned} x(t) * y(t) &= y(t) * x(t) = \int_{-\infty}^{+\infty} y(\tau) x(t-\tau) d\tau \\ &= \int_{-\infty}^{+\infty} \delta(\tau+2) 2e^{2(t-\tau)} d\tau \\ &\quad \uparrow \\ &\quad \text{fires at } \tau = -2, \text{ "sifting"} \\ &= 2e^{2(t+2)} \end{aligned}$$

Question

$$x(t) = u(t)e^{-2t}$$

$$h(t) = u(t-1)$$

$$x(t) * h(t) = y(t)$$

graph $x(t)$ and $h(t)$

Solve $y(t)$ by equation

Answer

$x(t)$ $h(t)$ $y(t) = \int_{-\infty}^{+\infty} x(\tau) h(t-\tau) d\tau$

$y(t) = \int_{-\infty}^{+\infty} u(\tau) e^{-2\tau} u(t-\tau-1) d\tau$

$= \int_0^{t-1} e^{-2\tau} d\tau = -\frac{1}{2} e^{-2\tau} \Big|_0^{t-1} = \frac{1}{2} [1 - e^{-2(t-1)}]$

for $t \geq 1$ integration is valid.
 $y(t) = 0$ for $t < 1$

$y(t) = u(t) \frac{1}{2} [1 - e^{-2(t-1)}]$

Question

$$x(t) = -u(t) \cos(t)$$

$$h(t) = \delta(t - \pi)$$

$$\text{Solve for } y(t) = x(t) * h(t)$$

mathematically.

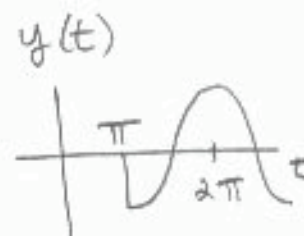
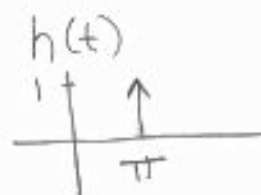
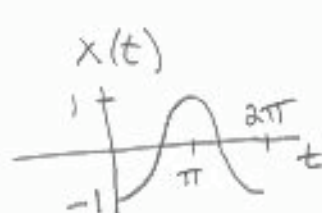
Then graph $x(t)$, $h(t)$, and $y(t)$
label all axes

Answer

$$y(t) = \int_{-\infty}^{+\infty} -u(\tau) \cos(\tau) \delta(t - \pi - \tau) d\tau$$

↑
fires at $\tau = t - \pi$

$$= -u(t - \pi) \cos(t - \pi)$$



Question

$$x(t) = -u(t)$$

$$h(t) = u(t) 6t$$

Graph $x(t)$ and $h(t)$

$$\text{Solve } y(t) = x(t) * h(t)$$

mathematically

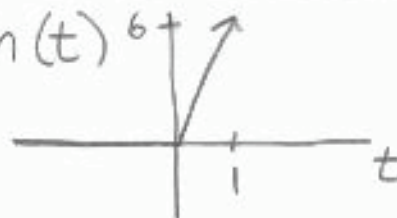
Then solve it graphically
and compare.

Answer

$x(t)$



$h(t) 6t$



$$y(t) = \int_{-\infty}^t x(\tau) h(t-\tau) d\tau = -6 \int_{-\infty}^t u(\tau) u(t-\tau) \tau d\tau$$

$$= -6 \int_0^t \tau d\tau = -3t^2$$

↑
for $t > 0$
otherwise $y = 0$

$y(t)$

