For the following periodic signal

$x[0] = -2 \quad x[1] = 6 \quad x[n] = x[n-2]$

what is N, the period?

using the inverse Fourier Matrix, F^{-1}, find the Fourier Series.

using the Fourier Matrix, F, recreate $x[n]$ from its fourier series.

$N = 2$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \underbrace{\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}_{F^{-1}} \begin{bmatrix} -2 \\ 6 \end{bmatrix}$$

$a_0 = 2 \leftarrow$ average value

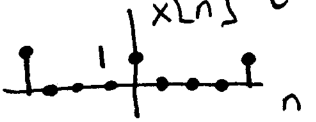
$a_1 = -4$

$$\begin{bmatrix} x[0] \\ x[1] \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}_{F} \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

$x[0] = -2$

$x[1] = 6$

For the following periodic signal



with period $N = 4$

use the inverse Fourier Matrix F^{-1} to find the Fourier series. Then use the Fourier Matrix F to recreate $x[n]$ from its Fourier series

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$a_0 = \frac{1}{4}$

$a_1 = \frac{1}{4}$

$a_2 = \frac{1}{4}$

$a_3 = \frac{1}{4}$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{bmatrix}$$

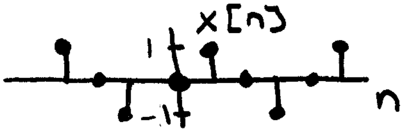
$x[0] = 1$

$x[1] = \frac{1}{4}(1 + j - 1 - j) = 0$

$x[2] = \frac{1}{4}(1 - 1 + 1 - 1) = 0$

$x[3] = \frac{1}{4}(1 - j - 1 + j) = 0$

For the following periodic signal



with period $N = 4$

use the inverse Fourier Matrix F^{-1} to find the Fourier series

Then use the Fourier Matrix F to recreate $x[n]$ from its Fourier Series.

What continuos function appears to have been sampled?

Why is a_0 the value that it is?

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$a_0 = 0$ ← average value

$$\left.\begin{aligned} a_1 &= -\frac{j}{2} \\ a_2 &= 0 \\ a_3 &= \frac{j}{2} \end{aligned}\right\} \sin\left(\frac{\pi n}{2}\right)$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 0 \\ -\frac{j}{2} \\ 0 \\ \frac{j}{2} \end{bmatrix}$$

$x[0] = 0$

$x[1] = 1$

$x[2] = 0$

$x[3] = -1$

Given the following Fourier Series coefficients for a discrete $x[n]$

$a_0 = -1$

$a_1 = 1$

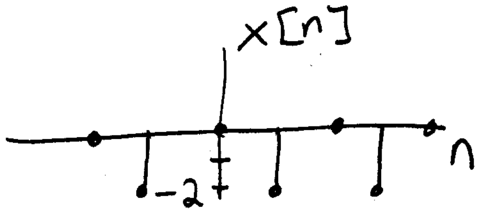
$a_k = a_{k-2}$, for all other k

What is the period N of $x[n]$, assuming $x[n] = x[n-N]$?

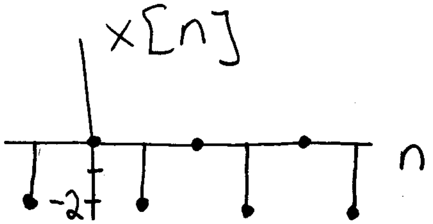
Draw a graph of $x[n]$

$N = 2$

$$\begin{bmatrix} x[0] \\ x[1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} \Rightarrow \begin{matrix} x[0] = 0 \\ x[1] = -2 \end{matrix}$$



For the following discrete periodic signal,



what is the <u>period N</u> and the <u>fundamental frequency ω_0</u> ?

(assume a sampling frequency $\omega_s = 2\pi$)

Find the coefficients a_K of the Fourier Series. How many independend a_K terms are there?

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ -2 \end{bmatrix} \Rightarrow \begin{matrix} a_0 = -1 \\ a_1 = 1 \end{matrix}$$

$N = 2$ $\quad \omega_0 = \pi$

2 independent terms

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

DTFS7

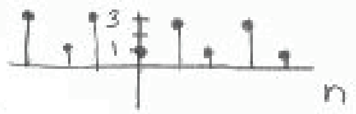
Problem Type Acronym

Name

ID #

Question

Given the following periodic $x[n]$



(A) what is the period N?

(B) How many independent terms a_k do you expect in the Fourier Series?

(C) Is $x[n]$ odd, even, or neither?

(D) Do you expect all a_k to be real or imaginary? (Pick one)

(E) What is the F^{-1} matrix in this case?

(F) Solve for all independent a_k.

Answer

(A) 2

(B) 2

(C) even

(D) real

(E) $F^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

(F) $\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$a_0 = 2 \quad a_1 = -1$