Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FS1

Problem Type Acronym

Name

ID #

Question

$x(t) = 2 - \cos(2t)$

find ω_0 and T_0

find all non-zero a_k

Answer

$\omega_0 = 2$

$T_0 = \frac{2\pi}{\omega_0} = \pi$

$a_0 = 2$

$a_1 = a_{-1} = -\frac{1}{2}$

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FS2

Problem Type Acronym

Name

ID #

Question

$$x(t) = 2 + \sum_{n=-\infty}^{+\infty} \delta(t-4n)$$

Find ω_0

what is the Fourier Series?

Answer

$$T_0 = 4 \qquad \omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$$

$$2 \xleftrightarrow{FS} a_0 = 2$$

$$\sum_{n=-\infty}^{+\infty} \delta(t-4n) \xleftrightarrow{FS} a_k = \frac{1}{T_0} = \frac{1}{4}$$

+

$$x(t) \xleftrightarrow{F} a_0 = 2\tfrac{1}{4}, \quad a_k\big|_{k\neq 0} = \frac{1}{4}$$

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FS3

Problem Type Acronym

Name

ID #

Question

Find the Fourier Series (all non-zero a_k) and specify ω_0 for

$$x(t) = \pi + \cos\left(2t - \frac{\pi}{4}\right)$$

Answer

$\omega_0 = 2$

$a_0 = \pi$

$a_1 = \frac{1}{2\sqrt{2}}(1-j)$

$a_{-1} = \frac{1}{2\sqrt{2}}(1+j)$

$$\cos\left(2t - \frac{\pi}{4}\right) = \frac{e^{j(2t-\frac{\pi}{4})} + e^{-j(2t-\frac{\pi}{4})}}{2} =$$

$$\frac{e^{-\frac{\pi}{4}} e^{j2t}}{2} + \frac{e^{\frac{\pi}{4}} e^{-j2t}}{2} =$$

$$\underbrace{\frac{1}{2\sqrt{2}}(1-j)e^{j2t}}_{k=1} + \underbrace{\frac{1}{2\sqrt{2}}(1+j)e^{-j2t}}_{k=-1}$$

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FS4

Problem Type Acronym | Name | ID #

Question

Compute the Fourier series of

$$x(t) = -3 + \cos(\pi t) - 2\sin(2\pi t)$$

what is ω_0?

what is the average power?

(use Parseval's relation)

Answer

$a_0 = -3$

$a_1 = \frac{1}{2} \quad a_{-1} = \frac{1}{2}$

$a_2 = j \quad a_{-2} = -j$

$\omega_0 = \pi$

$$P = \sum_{k=-\infty}^{\infty} |a_k|^2 = |-3|^2 + |\tfrac{1}{2}|^2 + |\tfrac{1}{2}|^2 + |j|^2 + |-j|^2$$

$$= 9 + \frac{1}{4} + \frac{1}{4} + 1 + 1$$

$$= 11\frac{1}{2}$$

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FS5

Problem Type Acronym | Name | ID #

Question

The Fourier series of $x(t)$ is

$$a_0 = 1$$
$$a_1 = 1 + j$$
$$a_{-1} = 1 - j$$

What is $x(t)$ if $\omega_0 = 2\pi$?

What is the Fourier series of $\frac{dx(t)}{dt}$?

(call it b_k)

Answer

$$x(t) = 1 + 2\cos(2\pi t) - 2\sin(2\pi t)$$

Fourier series of $\frac{dx(t)}{dt}$ is

$$b_k = jk\omega_0 a_k$$
$$b_0 = 0$$
$$b_1 = 2\pi(j-1)$$
$$b_{-1} = 2\pi(-j-1)$$

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FS6
Problem Type Acronym

Name

ID #

Question

Given the following Fourier series

$a_0 = 6 \quad a_1 = 2 + j \quad \omega_0 = 10$

$a_{-1} = 2 - j$

Find the time domain signal $x(t)$.

Find $\frac{dx(t)}{dt}$ and its Fourier Series a'_K.

What is the average value of $\frac{dx(t)}{dt}$?

Answer

$x(t) = 6 + 4\cos(10t) - 2\sin(10t)$

$a'_K = jK\omega_0 a_K$

$a'_0 = 0 \quad a'_1 = 20j - 10$

$a'_{-1} = -20j - 10$

$\frac{dx(t)}{dt} = -40\sin(10t) - 20\cos(10t)$

average value of $\frac{dx(t)}{dt}$ is 0

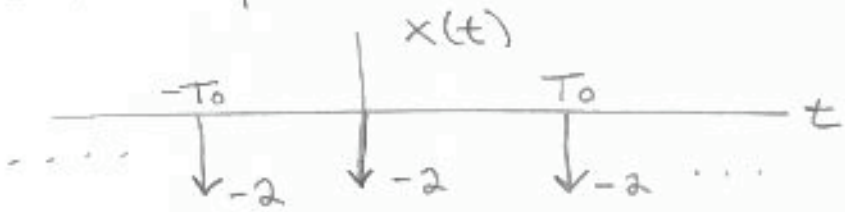
Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FS7

Problem Type Acronym | Name | ID #

Question

$x(t)$ is periodic and shown here



Write an equation for $x(t)$.
What is its Fourier series a_k?
Is $x(t)$ even or odd, and what does this imply about its Fourier series?

Answer

$$x(t) = \sum_{n=-\infty}^{+\infty} -2\delta(t - nT_0)$$

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}\,dt = -\frac{2}{T_0}\int_{-\frac{T_0}{2}}^{+\frac{T_0}{2}} \delta(t)\,dt$$

$$a_k = -\frac{2}{T_0}$$

$x(t)$ is even $\Rightarrow$ a_k is real

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FS8
Problem Type Acronym

Name

ID #

Question

$x(t) = 3 - \cos(2\pi t) + 2\sin(4\pi t)$

Find all non-zero terms of the Fourier Series a_k.

What is the average value of $x(t)$?

What is the fundamental frequency ω_0 and period T_0

What is the average power of $x(t)$, using Parseval's Relation?

Answer

$a_0 = 3$

$a_1 = a_{-1} = -\frac{1}{2}$

$a_2 = -j \quad a_{-2} = j$

average value is 3

$\omega_0 = 2\pi \qquad T_0 = 1$

$$\text{average power} = \sum_{k=-\infty}^{+\infty} |a_k|^2 =$$

$$|3|^2 + |-\tfrac{1}{2}|^2 + |-\tfrac{1}{2}|^2 + |-j|^2 + |j|^2 = 11\tfrac{1}{2}$$

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FS9

Problem Type Acronym | Name | ID #

Question

For

$$x(t) = -4 + \cos(2t + \pi)$$

what is the fundamental freq. ω_0? radians/sec.

what is the fundamental period T_0? sec.

Find all non zero terms in the Fourier Series a_k

is $a_k = a_{-k}^*$ in all cases?

Answer

$\omega_0 = 2$ radians/sec

$T_0 = \frac{2\pi}{\omega_0} = \pi$ seconds

$$x(t) = -4 + \frac{e^{j(2t+\pi)} + e^{-j(2t+\pi)}}{2}$$

$$= -4 + \underbrace{e^{j\pi}}_{-1}\left[\frac{e^{j2t} + e^{-j2t}}{2}\right]$$

$a_0 = -4 \quad a_1 = -\frac{1}{2} \quad a_{-1} = -\frac{1}{2}$

yes $a_k = a_{-k}^*$

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FS 10

Problem Type Acronym

Name

ID #

Question

given

$$x(t) = 2 + 3\cos\left(8\pi t + \frac{\pi}{4}\right)$$

What is the period T_0 and fundamental frequency ω_0 ?

Specify all non-zero Fourier Series coefficients a_k, separating the real and imaginary components of each coeffient. Is a_{-k} the complex conjugate of a_k in each case ?

Answer

$$T_0 = \frac{1}{4} \qquad \omega_0 = 8\pi$$

$$a_0 = 2$$

$$3\cos\left(8\pi t + \frac{\pi}{4}\right) = 3\left[\frac{e^{j(8\pi t + \frac{\pi}{4})} + e^{-j(8\pi t + \frac{\pi}{4})}}{2}\right]$$

$$= \underbrace{\frac{3}{2}e^{j\frac{\pi}{4}}}_{a_1} e^{j8\pi t} + \underbrace{\frac{3}{2}e^{-j\frac{\pi}{4}}}_{a_{-1}} e^{-j8\pi t}$$

$$\left.\begin{aligned} a_1 &= \frac{3}{\sqrt{2}} + j\frac{3}{\sqrt{2}} \\ a_2 &= \frac{3}{\sqrt{2}} - j\frac{3}{\sqrt{2}} \end{aligned}\right\} \text{yes, they are complex conjugates}$$

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FS 11

Problem Type Acronym | Name | ID #

Question

The Fourier series of $x(t)$ is

$a_0 = 6$

$a_1 = 1 + \frac{j}{2}$ $\quad a_{-1} = 1 - \frac{j}{2}$

$a_2 = -3$ $\quad a_{-2} = -3$

(A) what is $x(t)$, if $\omega_0 = 4$

(B) what is the Fourier Series of $\frac{dx(t)}{dt}$?

(C) what is the Fourier Series of $2x(t-\frac{\pi}{2})$?

Answer

(A) $x(t) = 6 + 2\cos(4t) - \sin(4t) - 6\cos(8t)$

(B) $\frac{dx(t)}{dt} \overset{Fs}{\longleftrightarrow} jk\omega_0 a_k$

$a_0 = 0$

$a_1 = 4j - 2$ $\quad a_{-1} = -4j - 2$

$a_2 = -24j$ $\quad a_2 = 24j$

(C) $x(t-2) \overset{Fs}{\longleftrightarrow} e^{-jk\omega_0} a_k$

$a_0 = 12$

$a_1 = 2 + j$ $\quad a_{-1} = 2 - j$

$a_2 = -3$ $\quad a_{-2} = -3$

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FS12

Problem Type Acronym | Name | ID #

Question

Given $x(t) = \sin(t/2\pi)$

$y(t) = \cos(t/2\pi)$

(A) what is the fundamental freq. ω_0

(B) what is the fundamental period T_0

(C) find Fourier Series of $x(t)$ and $y(t)$

(D) what is the average power of each signal (use Fourier Series)

(E) should shifting a signal change its average power?

Answer

(A) $\omega_0 = 1/2\pi$

(B) $T_0 = 1$

(C) $x(t) \overset{F_s}{\longleftrightarrow}$ $a_1 = -j/2$, $a_{-1} = +j/2$

$y(t) \overset{F_s}{\longleftrightarrow}$ $b_1 = 1/2$, $b_{-1} = 1/2$

(D) power of $x(t) = (-j/2)^2 + (j/2)^2 = \frac{1}{2}$

power of $y(t) = (\frac{1}{2})^2 + (\frac{1}{2})^2 = \frac{1}{2}$

(E) shifting should <u>not</u> change it.

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FS13

Problem Type Acronym | Name | ID #

Question

given the Fourier Series of $x(t)$ is

$a_0 = -2 \quad a_1 = 1+j \quad a_{-1} = 1-j$

$a_k = 0$ all other k, $\omega_0 = 4\pi$

(A) what is $x(t)$

(B) what is the Fourier Transform $X(\omega)$

(C) is $x(t)$ even, odd, or neither.

(D) is $x(t)$ periodic and if so, what is its period?

(E) What is the average power of $x(t)$?

Answer

(A) $x(t) = -2 + 2\cos 4\pi t - 2\sin 4\pi t$

(B) $X(\omega) = -2\pi\delta(\omega) + (1+j)\pi\delta(\omega - 4\pi) - (1-j)\pi\delta(\omega + 4\pi)$

(C) neither

(D) yes, $T_0 = \frac{2\pi}{\omega_0} = \frac{1}{2}$

(E) $\sum_{k=-\infty}^{+\infty} |a_k|^2 = |-2|^2 + |1+j|^2 + |1-j|^2$

$= 4 + 2 + 2 = 8$