Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FT 1
Problem Type Acronym

Name

ID #

Question

$x(t) = \delta(t) - \delta(t-1)$

graph the real and imaginary parts of $X(\omega)$. Where will $|X(\omega)| = 0$ and why?

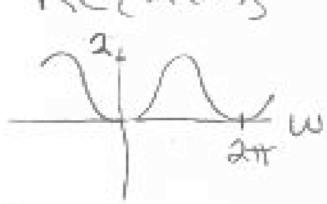
Answer

$X(\omega) = 1 - \underbrace{e^{-j\omega}}$ ← delayed by 1

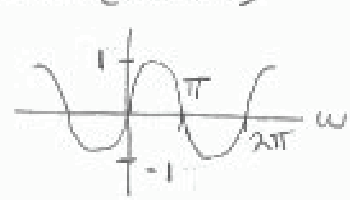
euler's ident. $\Rightarrow e^{-j\omega} = e^{j(-\omega)} = \cos(\omega) - j\sin(\omega)$

So, $X(\omega) = (1-\cos\omega) + j\sin\omega$

$Re\{X(\omega)\}$



$Im\{X(\omega)\}$



$|X(\omega)| = 0$ at $\omega = 0, 2\pi, 4\pi, \ldots$

Shifting by multiple full cycles and subtracting yields Ø output.

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FT 2

Problem Type Acronym | Name | ID #

Question

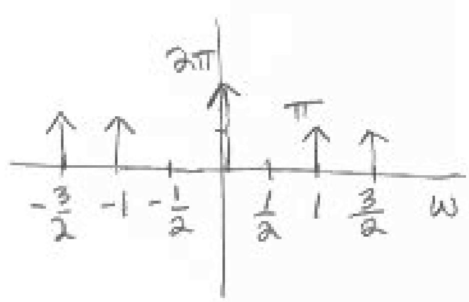
$x(t) = 1 + \cos(t) - \sin(t) + \cos(\frac{3}{2}t)$

graph $Re\{X(\omega)\}$ and $Im\{X(\omega)\}$

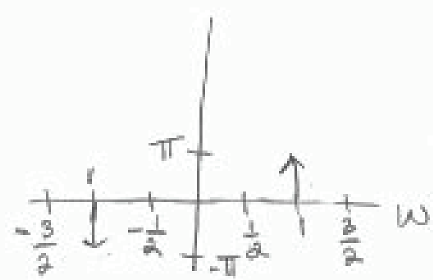
what is the fundamental frequency ω_0?

Answer

$Re\{X(\omega)\}$



$Im\{X(\omega)\}$



$\omega_0 = \frac{1}{2}$, even though there is no power there

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FT3
Problem Type Acronym

Name

ID #

Question

given $x(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT_0) \xleftrightarrow{Fs} a_K = \frac{1}{T_0}$

and $T_0 = 2\pi$

find w_0

graph $x(t - \frac{\pi}{2})$

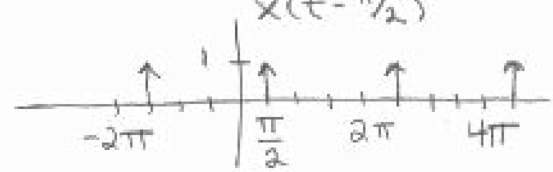
write the equation for its Fourier series b_K and graph the real and imaginary parts of b_K as a function of K

Does $|a_K| = |b_K|$?

Answer

$\omega_0 = \frac{2\pi}{T_0} = 1$

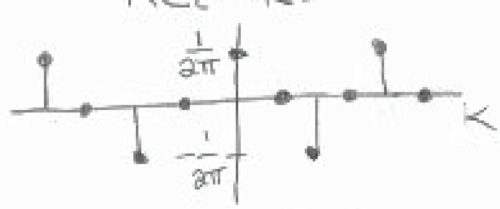
$x(t-\pi/2)$



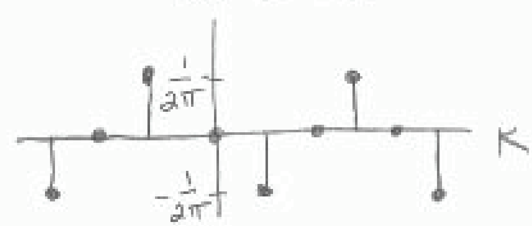
$$\boxed{x(t-\tau) \xleftrightarrow{Fs} e^{-jk\omega_0\tau} a_K}$$

$$b_K = a_K e^{-jk\omega_0\tau} = \frac{1}{2\pi} e^{-jK\frac{\pi}{2}}$$

$Re\{b_K\}$



$Im\{b_K\}$



yes, $|a_K| = |b_K|$. Only phase has changed.

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FT4
Problem Type Acronym

Name

ID #

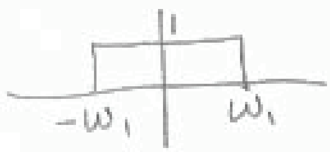
Question

given a system

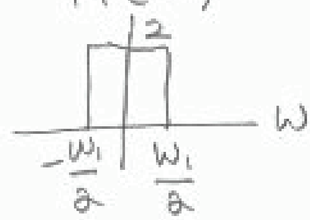
$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

where

$X(\omega)$



$H(\omega)$



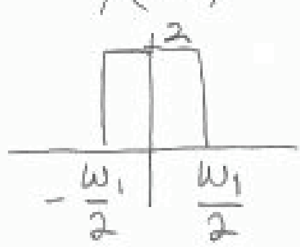
sketch $Y(\omega)$

Answer

$$x(t) * h(t) = y(t)$$

$$X(\omega) H(\omega) = Y(\omega)$$

$Y(\omega)$



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FT5

Problem Type Acronym

Name

ID #

Question

what is the Fourier Transform of

$$x(t) = -2\delta(t-3) \quad ?$$

Show that $|X(\omega)|$ is independent of ω.

Answer

$$\delta(t) \xleftrightarrow{F} 1$$

$$-2\delta(t) \xleftrightarrow{F} -2 \quad \text{(linear)}$$

$$-2\delta(t-3) \xleftrightarrow{F} -2e^{-j3\omega} = X(\omega)$$

because $\boxed{x(t-\tau) \xleftrightarrow{F} e^{-j\omega\tau} X(\omega)}$

$$|X(\omega)| = |-2e^{-j3\omega}| = |-2|\underbrace{|e^{-j3\omega}|}_{1} = 2$$

a delay line with a gain of -2.

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FT6 — Problem Type Acronym

Heather Greco — Name

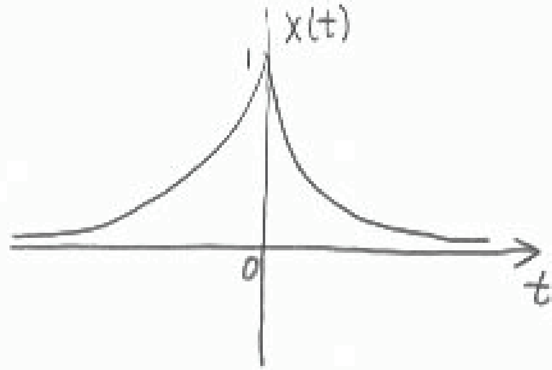
30 — ID #

Question

Find the Fourier transform of the signal

$x(t) = e^{-a|t|}, \quad a > 0$

x(t) can be written as $e^{-a|t|} = \begin{cases} e^{-at} & t > 0 \\ e^{at} & t < 0 \end{cases}$



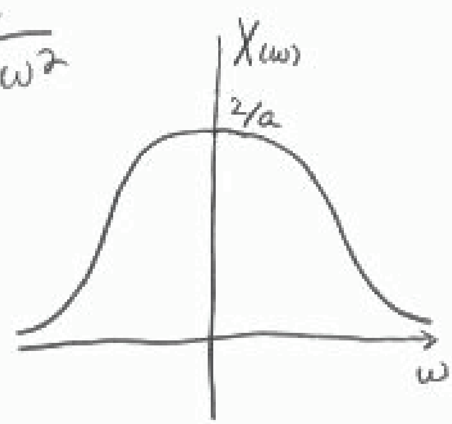
Answer

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt = \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \frac{2a}{a^2+\omega^2}$$

$$e^{-a|t|} \overset{F}{\longleftrightarrow} \frac{2a}{a^2+\omega^2}$$



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FT 7

Problem Type Acronym

Name

ID #

Question

Find the Fourier Series of

$$x(t) = -4 + \cos(3t) + \sin(3t)$$

what is the fundamental frequency ω_0 ?

What is the Fourier Transform of $x(t)$?

Answer

$a_0 = -4$

$a_1 = \frac{1}{2}(1-j)$

$a_{-1} = \frac{1}{2}(1+j)$

$\omega_0 = 3$

$$X(\omega) = -8\pi\delta(\omega) + \pi(1-j)\delta(\omega-3) + \pi(1+j)\delta(\omega+3)$$

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

Problem Type Acronym: FT8

Name

ID #

Question

$$x(t) = e^{j4t}$$

$$y(t) = \frac{x(t) + [x(t)]^*}{2}$$

← complex conjugate

give the equation for, and draw a graph of, the Real and imaginary parts of the fourier transform $Y(\omega)$

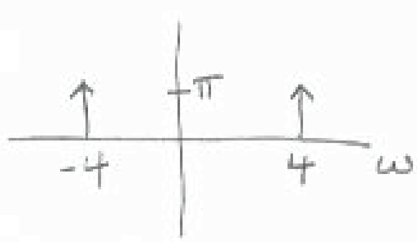
Answer

Since $[e^{j4t}]^* = e^{-j4t}$

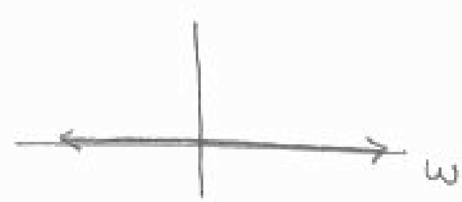
$$y(t) = \frac{e^{j4t} + e^{-j4t}}{2} = \cos(4t)$$

$$Y(\omega) = \pi[\delta(\omega-4) + \delta(\omega+4)]$$

$Re\{Y(\omega)\}$



$Im\{Y(\omega)\}$



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FT9

Problem Type Acronym

Name

ID #

Question

Find the fourier transform $Y(\omega)$ for

$$x(t) = 1 + e^{-t}u(t)$$

Answer

$$1 \xleftrightarrow{F} 2\pi\delta(\omega)$$

$$e^{-t}u(t) \xleftrightarrow{F} \int_{-\infty}^{+\infty} e^{-t}u(t)e^{-j\omega t}dt =$$

$$\int_0^{\infty} e^{-(1+j\omega)t}dt = -\frac{1}{1+j\omega}e^{-(1+j\omega)t}\Big|_0^{\infty} =$$

$$\frac{1}{1+j\omega}$$

$$X(\omega) = 2\pi\delta(\omega) + \frac{1}{1+j\omega}$$

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FT10

Problem Type Acronym | Name | ID #

Question

The Fourier transform of $x(t)$ is

$$X(\omega) = 2\pi\delta(\omega)$$

what is $x(t)$?

If $x(t)$ is put through a system which delays by one second to yield $y(t)$, what is $y(t)$?

$$x(t) \longrightarrow \boxed{\delta(t-1)} \longrightarrow y(t)$$

Answer

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} 2\pi\delta(\omega)e^{-j\omega t}\,dt = 1$$

if $h(t) = \delta(t-1)$

$H(\omega) = e^{-j\omega}$

$Y(\omega) = 2\pi\delta(\omega)e^{-j\omega} = 2\pi\delta(\omega)$

$y(t) = 1$

Delaying $x(t) = 1$ does not change it at all.

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FT 11

Problem Type Acronym

Name

ID #

Question

If $x(t) = 2\delta(t)$

what is the Fourier Transform $X(\omega)$ of $x(t)$? Show it using

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt.$$

hint: sifting!

what is the Fourier Transform of $x(t-1)$?

Answer

$$X(\omega) = \int_{-\infty}^{+\infty} 2\delta(t) e^{-j\omega t} dt = 2$$

Since $x(t-\tau) \xleftrightarrow{F} e^{-j\omega\tau} X(\omega)$

$$x(t-1) \xleftrightarrow{F} 2e^{-j\omega}$$

$(\tau = 1)$

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FT12

Problem Type Acronym

Name

ID #

Question

given the system

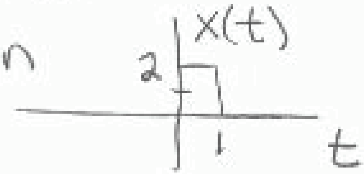
$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

where $h(t) = -\delta(t+1)$

Specify $H(\omega)$

$|H(\omega)|$ is a constant value. What is that value? (simplify it)

Given



Sketch $y(t)$ labeling axes.

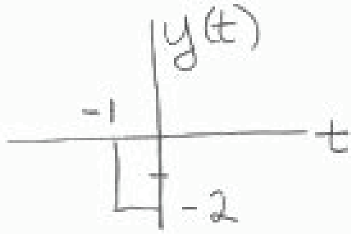
Answer

$$H(\omega) = \int_{-\infty}^{+\infty} \underbrace{-\delta(t+1)}_{} e^{-j\omega t}\, dt = -e^{j\omega}$$

sifting: fires when $t = -1$

$|H(\omega)| = 1$

convolve $x(t)$ with $h(t)$.



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FT13

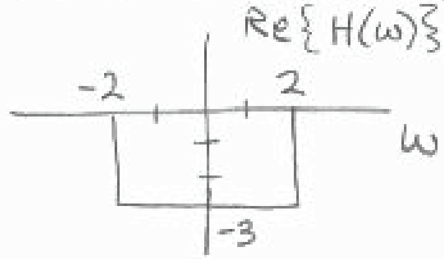
Problem Type Acronym

Name

ID #

Question

Given a system with impulse response $h(t)$ whose Fourier Transform $H(\omega)$ is



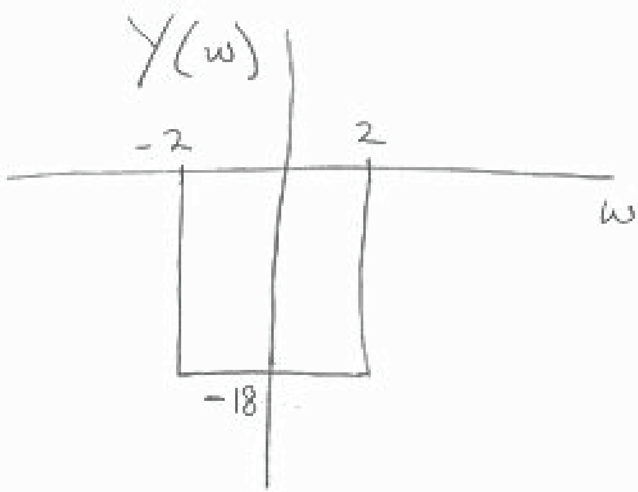
$$\mathrm{Im}\{H(\omega)\} = 0$$

Given an input signal $x(t)$ with Fourier Transform $X(\omega) = 6$

Sketch the Fourier Transform $Y(\omega)$ of the output signal.

Answer

Since $Y(\omega) = X(\omega)\,H(\omega)$



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FT14

Problem Type Acronym | Name | ID #

Question

The impulse response of a system is

$$h(t) = u(t)$$

(A) Find its Fourier Transform $H(\omega)$.

(B) Since $x(t) * h(t) = y(t) \Rightarrow X(\omega)H(\omega) = Y(\omega)$

What does this system do?

1) Delay
2) Integrate
3) Differentiate
4) Amplify

Answer

(A) $$H(\omega) = \int_{-\infty}^{+\infty} h(t) e^{-j\omega t} dt = \int_{-\infty}^{+\infty} u(t) e^{-j\omega t} dt$$

$$= \int_{0}^{+\infty} e^{-j\omega t} dt = -\frac{1}{j\omega} e^{-j\omega t} \Big|_{0}^{\infty} = \frac{1}{j\omega}$$

(B) integrate

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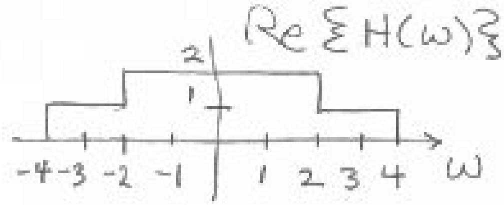
FT15
Problem Type Acronym

Name

ID #

Question

a system has a frequency response



with the imaginary part $Im\{H(\omega)\}=0$

given an input $x(t) = 2 - \sin t + \cos 3t$

A) what is the Fourier Transform, $X(\omega)$?

B) what is the output signal, $y(t)$?

C) what is its Fourier Transform, $Y(\omega)$?
(hint: $Y(\omega) = X(\omega)H(\omega)$)

Answer

A) $X(\omega) = 4\pi\delta(\omega) + \pi\delta(1-\omega) - \pi\delta(1+\omega) + \pi\delta(3-\omega) - \pi\delta(3+\omega)$

B) $4 - 2\sin t + \cos 3t$

C) $8\pi\delta(\omega) + 2\pi\delta(1-\omega) - 2\pi\delta(1+\omega) + \pi\delta(3-\omega) - \pi\delta(3+\omega)$

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

FT16

Problem Type Acronym | Name | ID #

Question

To find the fourier transform $H(\omega)$ of the impulse response $h(t)$ of a system

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

which integrates, that is,

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

(A) Solve for the impulse response $h(t)$

(B) find its Fourier Transform $H(\omega)$

Answer

(A) $h(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau = u(t)$

(B) $X(\omega) = \int_{-\infty}^{+\infty} u(t) e^{-j\omega t}\,dt = \int_{0}^{+\infty} e^{-j\omega t}\,dt$

$$= -\frac{1}{j\omega} e^{-j\omega t}\Big|_{0}^{\infty} = \frac{1}{j\omega}$$