Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

LT1

Problem Type Acronym

Name

ID #

Question

what is the impulse response of a system defined by the following diff. eq.

$$\frac{d^2y(t)}{dt^2} - 3\frac{dy(t)}{dt} + 2y(t) = x(t) \quad ?$$

Plot the zeros + poles on the s-plane.

Is the system stable?

Answer

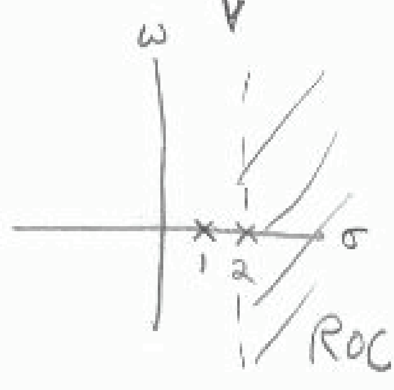
$$H(s) = \frac{1}{s^2 - 3s + 2} = \frac{1}{(s-1)(s-2)}$$

$$H(s) = \frac{c_1}{(s-1)} + \frac{c_2}{(s-2)}$$

$$c_1 = \left[H(s)(s-1)\right]_{s=1} = -1$$

$$c_2 = \left[H(s)(s-2)\right]_{s=2} = 1$$

$$h(t) = \left[-e^{t} + e^{2t}\right]u(t)$$



unstable

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

LT2

Problem Type Acronym | Name | ID #

Question

given

$$x(t) = e^{t}u(t) \longrightarrow \boxed{h(t) = \frac{d\delta(t)}{dt}} \longrightarrow y(t) = \frac{dx(t)}{dt}$$

(i.e. the system takes derivatives), graph $x(t)$; find and graph $y(t)$ show that $X(s)H(s) = Y(s)$

Answer

$$x(t) = e^{t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s-1} = X(s)$$

$$y(t) = \frac{dx(t)}{dt} = \delta(t) + e^{t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} 1 + \frac{1}{s-1} = Y(s)$$

$$h(t) = \frac{d\delta(t)}{dt} \overset{\mathcal{L}}{\longleftrightarrow} s = H(s)$$

$$X(s)H(s) \overset{?}{=} Y(s)$$

$$\frac{1}{s-1} \cdot s = 1 + \frac{1}{s-1}$$

$$\frac{s}{s-1} = \frac{s-1+1}{s-1} = \frac{s}{s-1} \quad \text{QED}$$

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Problem Type Acronym: LT3

Name:

ID #:

Question

what is the impulse response of a system defined by the following diff. eq.

$$\frac{d^2y(t)}{dt^2} + 4y(t) = \frac{dx(t)}{dt}$$

Plot the poles and zeros on the s-plane.

Answer

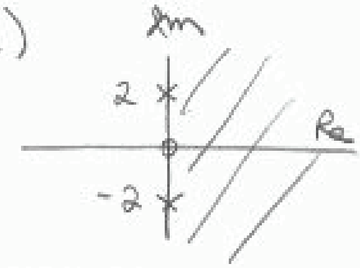
$$H(s) = \frac{s}{s^2+4} = \frac{s}{(s+2j)(s-2j)}$$

$$H(s) = \frac{C_1}{s+2j} + \frac{C_2}{s-2j}$$

$$C_1 = \left[H(s)(s+2j)\right]_{s=-2j} = \frac{-2j}{-4j} = \frac{1}{2}$$

$$C_2 = \left[H(s)(s-2j)\right]_{s=2j} = \frac{2j}{4j} = \frac{1}{2}$$

$$h(t) = \frac{1}{2}\left[e^{-2jt} + e^{+2jt}\right]u(t)$$



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Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

LT4

Problem Type Acronym | Name | ID #

Question

Find the Laplace transform $H(s)$ of

$$h(t) = u(t)\left[2 - e^{t}\right]$$

plot all poles + zeros on the s-plane showing the Region of Convergence.

Is it stable?

(Hint: Express $H(s)$ in rational form,

$$H(s) = \frac{N(s) \leftarrow \text{numerator}}{D(s) \leftarrow \text{denominator}}$$)

What is the corresponding differential equation for the system

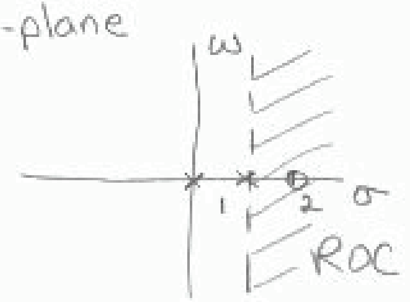
$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) \quad ?$$

Answer

Since $e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}$

$$u(t)\left[2e^{0t} - e^{t}\right] \overset{\mathcal{L}}{\longleftrightarrow} \frac{2}{s} - \frac{1}{s-1} = \frac{s-2}{(s)(s-1)}$$

s-plane



ROC does not contain the ω axis. Therefore system is UNSTABE

$$H(s) = -\frac{s-2}{s^2-s} \Rightarrow \frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} = \frac{dx(t)}{dt} - 2x(t)$$

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

LT5

Problem Type Acronym | Name | ID #

Question

Find the Laplace transform $H(s)$ of

$$h(t) = u(t)\left[e^{-3t}\cos(6t)\right]$$

plot all poles and zeros on the s-plane, showing the Region of Convergence.

Is it stable?

what is the corresponding differential equation of the system

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) \quad ?$$

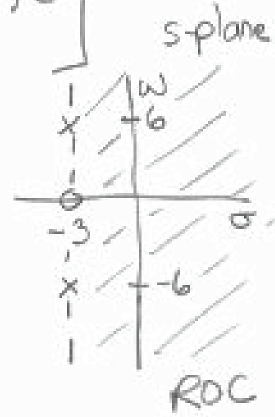
Answer

$$h(t) = u(t)e^{-3t}\left[\frac{e^{6jt} + e^{-6jt}}{2}\right]$$

$$= u(t)\frac{1}{2}\left[e^{(-3+6j)t} + e^{(-3-6j)t}\right]$$

$$H(s) = \frac{1}{2}\left[\frac{1}{s+3-6j} + \frac{1}{s+3+6j}\right]$$

$$H(s) = \left[\frac{s+3}{s^2+6s+45}\right]$$



$$\Downarrow$$

$$\frac{d^2y(t)}{dt} + 6\frac{dy(t)}{dt} + 45y(t) = \frac{dx(t)}{dt} + 3x(t)$$

Stable

ROC contains ω-axis

3

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

LT6

Problem Type Acronym

Name

ID #

Question

given the following differential equation

$$\frac{dy(t)}{dt} + \frac{y(t)}{dt} = x(t)$$ for a system

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

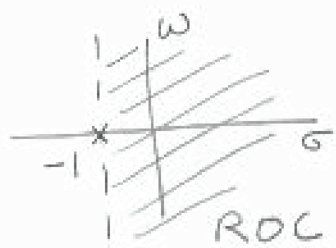
find $H(s)$ and $h(t)$

plot the poles and zeros on the s-plane

Is it stable?

Answer

$$H(s) = \frac{1}{s+1} \xleftrightarrow{\mathcal{L}} h(t) = u(t)e^{-t}$$



ROC contains ω-axis ⇒ stable

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

LT7
Problem Type Acronym

Name

ID #

Question

Given a system $H(s)$

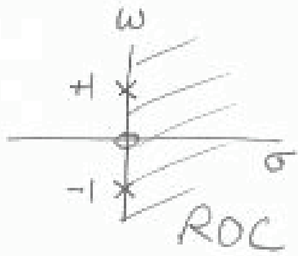
where the differential equation for $G(s)$ is $\frac{dy(t)}{dt} = x(t)$ and for $F(s)$ is $\frac{dy(t)}{dt} = x(t)$

Find $G(s)$, $F(s)$, $H(s)$, and the diff. eq. defining $H(s)$. Plot the poles + zeros. Is $H(s)$ stable?

Answer

$$G(s) = \frac{1}{s} \qquad F(s) = \frac{1}{s}$$

$$H(s) = \frac{G(s)}{1+F(s)G(s)} = \frac{\frac{1}{s}}{1+\frac{1}{s^2}} = \frac{s}{s^2+1}$$



$$\frac{d^2y(t)}{dt^2} + y(t) = \frac{dx(t)}{dt}$$

unstable

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LT8
Problem Type Acronym

Name

ID #

Question

given a system governed by the differential equation

$$\frac{d^2y(t)}{dt} + 3\frac{dy(t)}{dt} + 2y(t) = 2\frac{dx(t)}{dt} + x(t)$$

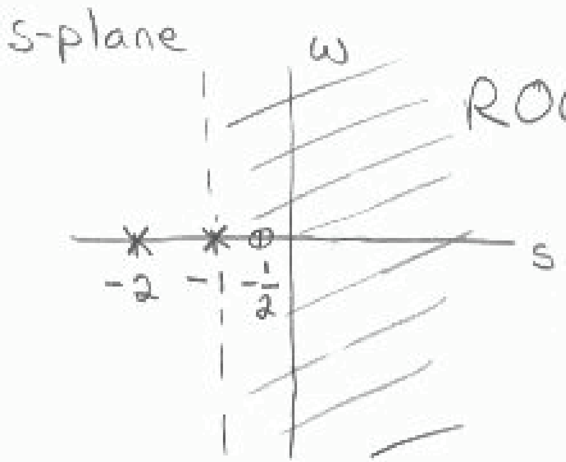
Solve for H(s), the Laplace Transform of the impulse response.

Plot the poles and zeros on the s-plane.

is the system stable?

Answer

$$H(s) = \frac{2s+1}{s^2+3s+2} = \frac{2s+1}{(s+2)(s+1)}$$



ROC contains ω - axis

⇓

stable

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

LT9		
Problem Type Acronym	Name	ID #

Question

Given a system

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

defined by the differential equation

$$2\frac{dy(t)}{dt} - 3y(t) = x(t)$$

Find the Laplace Transform for the system, $H(s)$, and the impulse response, $h(t)$.
Plot the poles and zeros of $H(s)$ on the s-plane. Is the system stable?

Answer

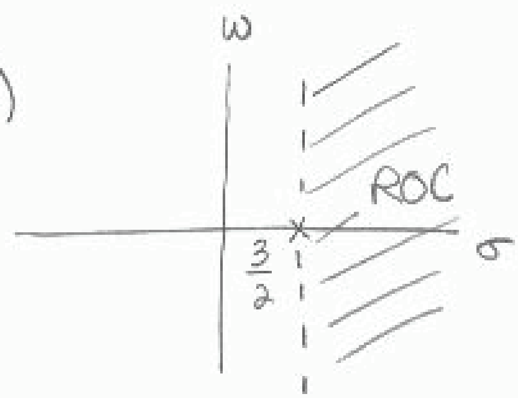
$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{2s-3} = \frac{1}{2}\left[\frac{1}{s-\frac{3}{2}}\right]$$

Since $e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}$

let $a = -\frac{3}{2}$

$$h(t) = \frac{1}{2}e^{\frac{3}{2}t}u(t)$$

unstable



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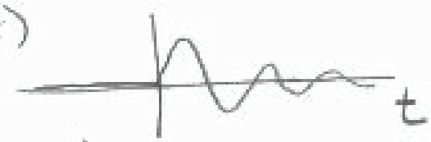
LT10

Problem Type Acronym — Name — ID #

Question

given a system with an impulse response $h(t) = \sin(2\pi t)e^{-t}u(t)$
Sketch $h(t)$ and state whether the system is stable (by inspection).
Find $H(s)$ and plot the poles and zeros on the s-plane.
Show that the Region of Convergence supports your previous conclusion about the system's stability.

Answer



stable

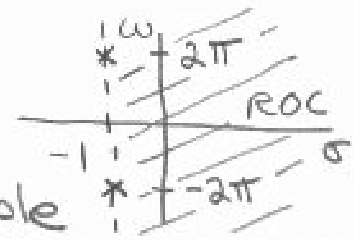
$$h(t) = \left[\frac{e^{j2\pi t} - e^{-j2\pi t}}{2j}\right] e^{-t}u(t)$$

$$h(t) = \frac{1}{2j}\left[e^{-(1-2\pi j)t} - e^{-(1+2\pi j)t}\right]u(t)$$

$$H(s) = \frac{1}{2j}\left[\frac{1}{s+1-2\pi j} - \frac{1}{s+1+2\pi j}\right]$$

$$H(s) = \frac{2\pi}{(s+1-2\pi j)(s+1+2\pi j)}$$

ROC contains ω-axis $\Rightarrow$ stable



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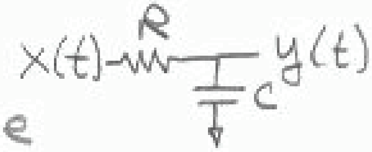
LT 11

Problem Type Acronym | Name | ID #

Question

The following RC circuit has an impulse response



$h(t) = \frac{1}{RC} e^{-t/RC} u(t)$. $R>0$, $C>0$.

(A) what is the Laplace transform $H(s)$?

(B) Plot all poles and zeros, as well as the region of convergence on the s-plane. Lable axes and key points.

(C) Is this system stable?

(D) Express the system as a differential equation of $x(t)$ and $y(t)$.

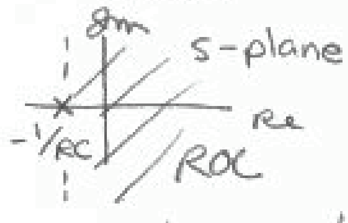
NOTE: $x(t)$ and $y(t)$ are voltages

Answer

(A) $e^{-at} u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}$

$a = 1/RC$ $\quad H(s) = \frac{1/RC}{s + 1/RC} = \frac{1}{sRC+1}$

(B)



(C) yes it is stable

(D) $x(t) = RC\frac{dy(t)}{dt} + y(t)$

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LT12

Problem Type Acronym | Name | ID #

Question

A resonant system has a diff. eq.

$$4\frac{d^2y(t)}{dt} + y(t) = x(t)$$

(A) Find the Laplace transform $H(s)$ of the impulse response.

(B) Draw any poles/zeros on the s-plane labeling axes and region of convergence.

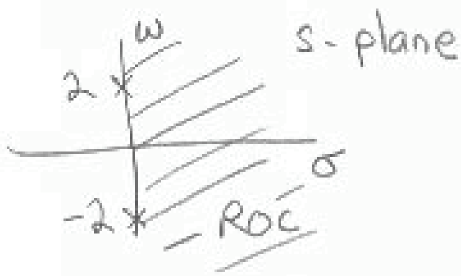
(C) At what frequency does it resonate?

(D) ____'s Law could govern such a physical system (ideally).

Answer

(A) $\dfrac{1}{4s^2+1}$

(B)



(C) $\omega = 2$

(D) Hooke (Newton)