

Question

Using phasors, prove that
$$\cos \alpha \cos \beta = \frac{1}{2} \cos(\alpha + \beta) + \frac{1}{2} \cos(\alpha - \beta)$$

Answer

$$\begin{aligned} & \frac{e^{j\alpha} + e^{-j\alpha}}{2} \cdot \frac{e^{j\beta} + e^{-j\beta}}{2} = \\ & \frac{e^{j(\alpha+\beta)} + e^{-j(\alpha+\beta)}}{4} + \frac{e^{j(\alpha-\beta)} + e^{-j(\alpha-\beta)}}{4} = \\ & \frac{1}{2} \cos(\alpha + \beta) + \frac{1}{2} \cos(\alpha - \beta) \end{aligned}$$

Question

Prove that

$$\sin^2 t = \frac{1}{2}(1 - \cos 2t)$$

using phasors

Answer

$$\begin{aligned}\sin^2 t &= \frac{e^{jt} - e^{-jt}}{2j} \cdot \frac{e^{jt} - e^{-jt}}{2j} \\&= \frac{e^{j2t} - e^0 - e^0 + e^{-j2t}}{-4} \\&= \frac{e^{j2t} + e^{-j2t} - 2}{-4} = \frac{1}{2}(1 - \cos 2t)\end{aligned}$$

Question

Using phasor representations
prove

$$\sin(2t) = 2\sin(t)\cos(t)$$

Answer

$$2 \left[\frac{e^{jt} - e^{-jt}}{2j} \right] \left[\frac{e^{jt} + e^{-jt}}{2} \right] =$$
$$\frac{e^{2jt} - e^{-2jt}}{2j} = \sin(2t)$$

Question

Prove the following using phasors

$$\sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) =$$

$$\sin(\alpha + \beta)$$

Answer

$$\begin{aligned} & \left(\frac{e^{j\alpha} - e^{-j\alpha}}{2j} \right) \left(\frac{e^{j\beta} + e^{-j\beta}}{2} \right) + \left(\frac{e^{j\alpha} + e^{-j\alpha}}{2} \right) \left(\frac{e^{j\beta} - e^{-j\beta}}{2j} \right) = \\ & \frac{e^{j(\alpha+\beta)} + \cancel{e^{j(\alpha-\beta)}} - \cancel{e^{j(\beta-\alpha)}} - e^{-j(\alpha+\beta)}}{4j} + \frac{e^{j(\alpha+\beta)} - \cancel{e^{j(\alpha-\beta)}} + \cancel{e^{j(\beta-\alpha)}} - e^{-j(\alpha+\beta)}}{4j} = \\ & \frac{e^{j(\alpha+\beta)} - e^{-j(\alpha+\beta)}}{2j} = \sin(\alpha + \beta) \end{aligned}$$

Question

Prove the following using phasors

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

Answer

$$\begin{aligned}\frac{e^{j2\alpha} + e^{-j2\alpha}}{2} &= 2 \left[\frac{e^{j\alpha} + e^{-j\alpha}}{2} \right] \left[\frac{e^{j\alpha} + e^{-j\alpha}}{2} \right] - 1 = \\ \frac{e^{j2\alpha} + e^{j0} + e^{j0} + e^{-j2\alpha}}{2} - 1 &= \\ \frac{e^{j2\alpha} + e^{-j2\alpha}}{2}\end{aligned}$$

Question

Prove the following
using phasors

$$\int \cos(\omega t) dt = \frac{1}{\omega} \sin(\omega t)$$

Answer

$$\cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\int \frac{e^{j\omega t}}{2} dt + \int \frac{e^{-j\omega t}}{2} dt =$$

$$\frac{1}{\omega} \cdot \frac{e^{j\omega t} - e^{-j\omega t}}{2j} = \frac{1}{\omega} \sin(\omega t)$$

Question

Prove the following
using phasors

$$\cos(s) \cos(t) = \frac{\cos(s+t) + \cos(s-t)}{2}$$

Answer

$$\begin{aligned} & \frac{\overbrace{\cos(s)}^{e^{js} + e^{-js}}}{2} \cdot \frac{\overbrace{\cos(t)}^{e^{jt} + e^{-jt}}}{2} = \\ & \frac{e^{j(s+t)} + e^{-j(s+t)}}{4} + \frac{e^{j(s-t)} + e^{-j(s-t)}}{4} = \\ & \frac{\cos(s+t)}{2} + \frac{\cos(s-t)}{2} \end{aligned}$$