

Question

Find the z transform for

$$x[n] = [9 + 3 \cdot 2^n] u[n]$$

what are the poles
and zeros?

Answer

$$\text{since } a^n u[n] \xleftrightarrow{z} \frac{z}{z-a}$$

$$9u[n] \xleftrightarrow{z} \frac{9z}{z-1}$$

$$3 \cdot 2^n u[n] \xleftrightarrow{z} \frac{3z}{z-2}$$

$$x[n] \xleftrightarrow{z} \frac{9z}{z-1} + \frac{3z}{z-2} =$$

$$\frac{9z^2 - 18z + 3z^2 - 3z}{(z-1)(z-2)} =$$

$$\frac{(12z - 21)z}{(z-1)(z-2)}$$

poles at 1, 2

zeros at 0, $\frac{21}{12}$

Question

given $x[n] = \left(\frac{1}{2}\right)^n u[n]$

Find the Z-transform.

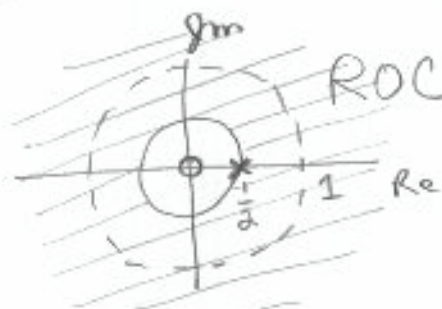
Plot the poles + zeros (if any) on the Z-plane.

Can the signal have a Fourier Transform performed on it (is it stable?)

Answer

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{+\infty} x[n] z^{-n} = \sum_{n=0}^{+\infty} \left(\frac{1}{2}\right)^n z^{-n} \\ &= \sum_{n=0}^{+\infty} \left(\frac{1}{2z}\right)^n = \frac{1}{1 - \frac{1}{2z}} = \frac{z}{z - \frac{1}{2}} \end{aligned}$$

$$\begin{aligned} \sum_{n=0}^{+\infty} \alpha^n &= \frac{1}{1-\alpha} \\ |\alpha| &< 1 \end{aligned}$$



ROC contains unit circle $\Rightarrow$ stable

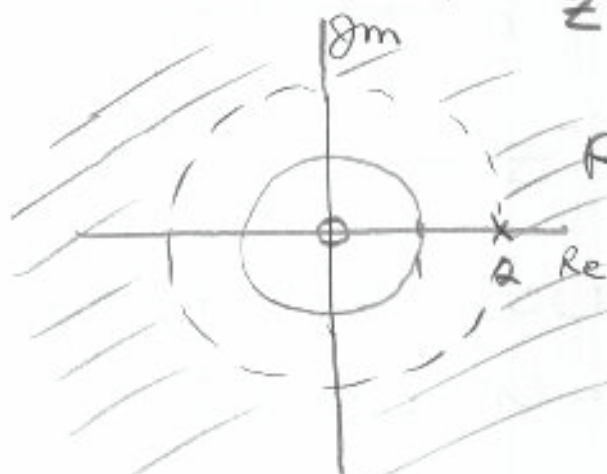
Question

Clearly a system with impulse response $h[n] = 2^n u[n]$ is not stable. What is its Z transform? Plot its poles and zeros on the Z plane. Does the ROC contain the unit circle?

Answer

$$a^n u[n] \xleftrightarrow{Z} \frac{z}{z-a}$$

$$a=2 \quad H(z) = \frac{z}{z-2}$$



ROC does not contain unit circle.

Use 0.7 mm mechanical pencil. Keep 0.25 inch from edge of box. Erase mistakes thoroughly.

ZT4

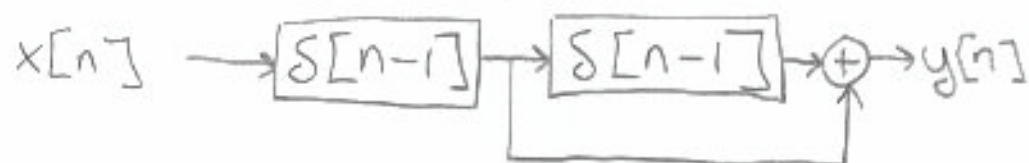
Problem Type Acronym

Name

ID #

Question

What is the impulse response $h[n]$ of the following system?



Is it IIR or FIR?

What is its Z transform $H(z)$?

Are there any poles (where $H(z) = \infty$)?

Is the system stable?

Answer

$$h[n] = \delta[n-1] + \delta[n-2]$$



$$H(z) = z^{-1} + z^{-2}$$

QUESTION OF POLE AT $z=0$
DISCUSS IN CLASS

Question

A system has an impulse response

$$h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$$

- (A) given that input $x[n]$ is the daily price of Apple stock, describe what the system does.
- (B) what is the z transform $H(z)$?
- (C) are there any poles?
- (D) is the system stable?
- (E) is it FIR or IIR?

Answer

- (A) Three day moving sum
- (B) $H(z) = 1 + z^{-1} + z^{-2}$
- (C) pole at $z = 0$
- (D) yes it is stable
- (E) FIR