BioE 1410 - Homework 4

For questions 1 - 3, use the following notation for the coefficients of the respective Fourier series.

$$x(t) \xleftrightarrow{Fs} a_k$$
$$y(t) \xleftrightarrow{Fs} b_k$$
$$h(t) \xleftrightarrow{Fs} c_k$$
$$z(t) \xleftrightarrow{Fs} d_k$$

1. Find all non-zero a_k and b_k for

$$x(t) = \sin(\omega_0 t) + \cos(2\omega_0 t) \qquad \text{and} \qquad y(t) = \cos(\omega_0 t) + \sin(2\omega_0 t)$$

Then, using Parseval's relation, show that the average power of $x(t)$ and $y(t)$ are the same.

2. Given $x(t)$ and $y(t)$ from question 1, find $z(t)$ in each of the following cases first by solving for d_k from a_k and b_k and building $z(t)$ from the Fourier series, and then by solving for $z(t)$ by simple calculus from $x(t)$ and $y(t)$. You should get the same results.

A. $z(t) = \dfrac{dx(t)}{dt}$ **B.** $z(t) = \dfrac{d[y(t) - 2x(t)]}{dt}$ **C.** $z(t) = \int y(t)dt$

3. Given a system with impulse response $h(t) = \delta(t-1)$ and a periodic $x(t) = x(t-2)$ with a Fourier series

$a_0 = 17$, $a_1 = 3$, $a_{-1} = 3$, $a_2 = 2j$, $a_{-2} = -2j$, $a_k = 0$ for all other k,

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

What is $x(t)$? Find the Fourier series of the output $y(t)$. What is $y(t)$?

Hint: $\boxed{x(t-\tau) \xleftrightarrow{Fs} e^{-jk\omega_0\tau} a_k}$ think about the phase shift of each harmonic.

4. Find all the non-zero a_k terms in the Fourier Series for the following signals. Also find the fundamental frequency ω_0 and the fundamental period T_0.

A. $x(t) = \sin(4\pi t)$

B. $x(t) = 4 - \cos(2\pi t) + 3\sin(6\pi t)$

C. $x(t) = \cos(4\pi t - \pi)$

5. What is the average power of each signal in question 4, using the Fourier series a_k and Parseval's relation?

6. Using the Fourier series you found in question 4A for $x(t) = \sin(4\pi t)$, let $y(t) = 2x(3t)$.

Using the fact that $x(\alpha t) \xleftrightarrow{Fs} a_k, \quad \alpha > 0$

Find the following:

A. All the non-zero b_k terms in the Fourier Series of $y(t)$

B. The fundamental period T_0 of $y(t)$

C. The fundamental frequency ω_0 of $y(t)$

D. The average power of $y(t)$

7. Find all non-zero a_k and for

$x(t) = 2 - \sin(\omega_0 t) + \cos(2\omega_0 t) - 2\sin(2\omega_0 t)$

Then, using Parseval's relation, find the average power of $x(t)$.
Sketch the real and imaginary parts of the Fourier Transform $X(\omega)$ labeling all axes and coordinates.
Is the total energy of $x(t)$ finite or infinite?

8. Given that $x(t) = \sin(2\pi t)$ and $h(t) = -2\partial(t)$, write equations for the Fourier transforms $X(\omega)$ and $H(\omega)$ and graph both spectra (real and/or imaginary parts, as needed) labeling all axes and coordinates. Write an equation for the output of the system $y(t) = x(t) * h(t)$ and its Fourier transform $Y(\omega)$, and graph $Y(\omega)$.

(9) Proove that $|X(\omega)|$ is always even for real $x(t)$

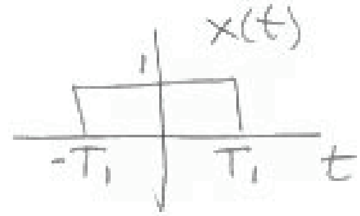
(10)

Proove that $\measuredangle X(\omega)$ is always odd for real $x(t)$

given $\operatorname{sinc}\lambda \triangleq \dfrac{\sin \pi\lambda}{\pi\lambda}$

(11) Prove that the Fourier Transform of

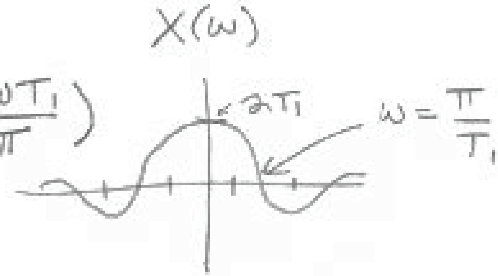
$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases}$$



is

$$X(\omega) = \frac{2\sin(\omega T_1)}{\omega} = 2T_1 \operatorname{sinc}\left(\frac{\omega T_1}{\pi}\right)$$

$\left(\lambda = \dfrac{\omega T_1}{\pi}\right)$



---

(12) $x(t) = 3\displaystyle\sum_{n=-\infty}^{+\infty} \delta(t-2n)$ $\quad$ n is integer

A. is $x(t)$ periodic?

B. find period T_0 and fundamental freq. ω_0

c. draw $x(t)$

d. solve for Fourier Series a_k.

e. draw a_k as function of k

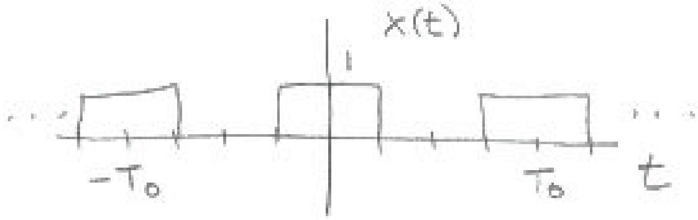
f. given $X(\omega) = \displaystyle\sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(\omega - k\omega_0)$

find Fourier Transform $X(\omega)$

g. draw $X(\omega)$

(13) given

$$x(t) = \begin{cases} 1, & |t| < \frac{T_0}{4} \\ 0, & \frac{T_0}{4} \le |t| \le \frac{T_0}{2} \\ x(t+T_0) = x(t) & \text{elsewhere} \end{cases}$$



Show that the Fourier Series is

$$a_K = \frac{1}{2} \operatorname{sinc}\left(\frac{K}{2}\right)$$

(hint: integrate from $-\frac{T_0}{4}$ to $\frac{T_0}{4}$, the only non-zero part of the period $-\frac{T_0}{2}$ to $\frac{T_0}{2}$)

(14) $x(t) \to \boxed{h_1(t)} \to \boxed{h_2(t)} \to y(t)$

given that $h_1(t)$ differentiates and $h_2(t)$ integrates, find $H_1(\omega)$, $H_2(\omega)$ and $H(\omega)$, where $h(t)$ is the overall system. What is $h(t)$ and why?

(15) a system that exhibits first order decay $h(t) = e^{-at}u(t)$, $a>0$ what is the spectrum $H(\omega)$? Does it favor high or low frequencies?

(16) Given $x(t) = \cos(3t) + 2\sin(3t)$ find $X(\omega)$ and $Y(\omega)$ if $y(t) = x(-t)$ (hint: since $x(t)$ is real, think complex conjugates). Explain in terms of cos and sin being even and odd respectively.