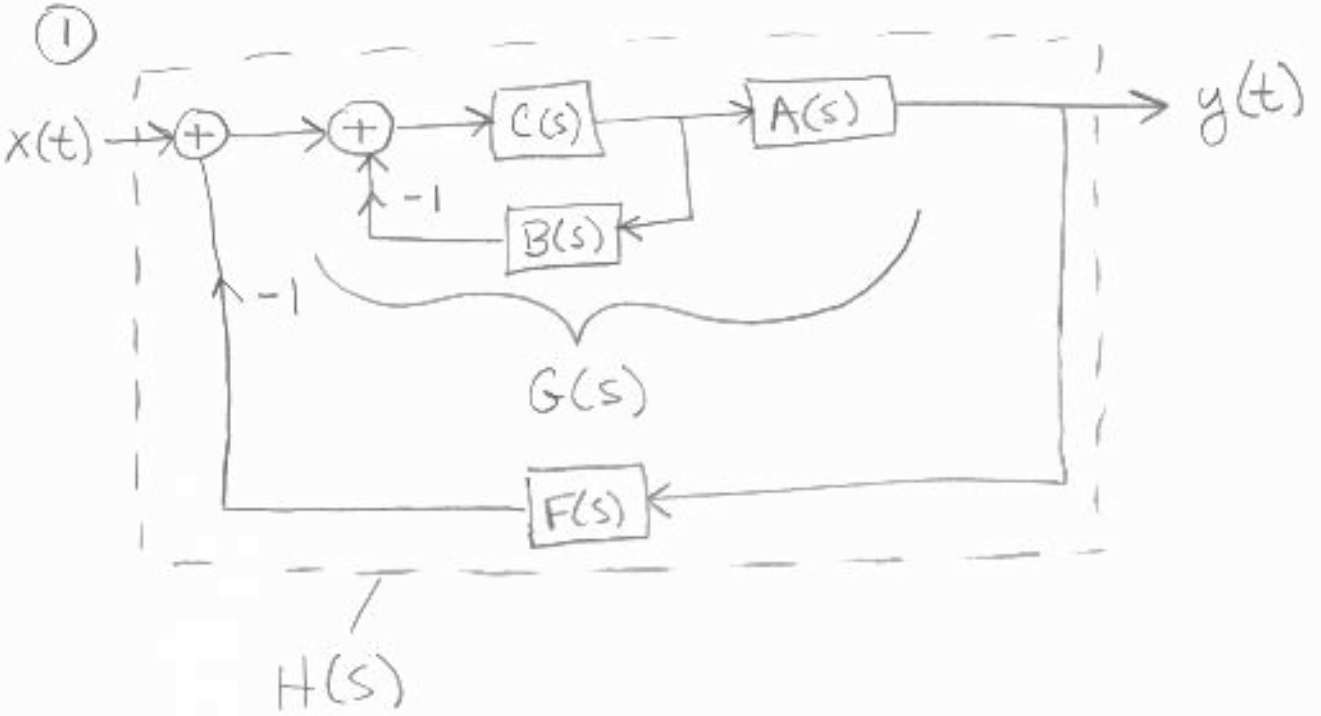
Homework 7

①



Using the standard formula for Closed loop gain, derive an expression for H(s) by first finding G(s).

Given the following Diff. Eq's for subsystems A, B, C, and F, find the Diff. Eq. for system H.

System

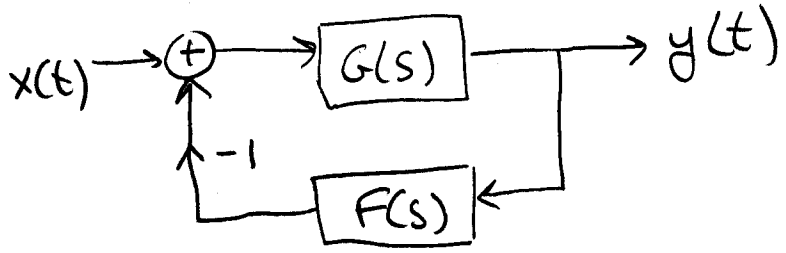
A $\frac{dy(t)}{dt} + 10\,y(t) = 2x(t)$

B $y(t) = 3\frac{dx(t)}{dt}$

C $\frac{dy(t)}{dt} + 10y(t) = \frac{dx(t)}{dt} + 5x(t)$

F $\frac{dy(t)}{dt} = x(t)$

(2)



The open loop impulse response $g(t) = \cos(t)u(t)$, that is, it rings forever.

What is $G(s)$?
Sketch its poles and zeros.

Using proportional feedback $F(s) = k$ derive an expression for the closed loop gain, $H(s)$ and find the value for k that puts both poles at -1, removing all oscillation.

(3) A wagon wheel with 8 spokes appears, in an old movie, to be spinning backwards at 1 cycle per second. The camera frame rate was 24 frames per second. Derive an expression for all possible actual speeds (cycles per second) for the wagon wheel.

④ For the following periodic functions, find the Fourier series, by first saying what N, the smallest period, is, then defining the inverse Fourier matrix, F^{-1}, and multiplying by it. Check your answer by defining the Fourier matrix, F, and mutiplying the vector of a_k values by it.

A) $x[0]=1,\ x[1]=0, x[2]=-1,\ x[3]=0$
$x[n]=x[n-4]$

B) $x[0]=0,\ x[1]=3,\ x[2]=0$
$x[n]=x[n-3]$

← HINT: KEEP MATRICES IN PHASOR NOTATION

C) $x[0]=3,\ x[1]=1$
$x[n]=x[n-2]$

D) $x[n]=5$, for all n

E) $x[0]=3,\ x[1]=0,\ x[2]=0$
$x[n]=x[n-3]$

Compare the Fourier Series for B) and E) and explain the relationship