BioE 1410 - Homework 1 - Answers

1. (10 pts.) Express each of the following complex numbers in Cartesian form $x + jy$, drawing a picture of each on the complex plane.

A. $\frac{1}{2}e^{j2\pi} = \frac{1}{2}e^{j0} = \frac{1}{2}e^{0} = \frac{1}{2}$

$x = \frac{1}{2} \qquad y = 0$

B. $-e^{(2-j\pi)} = -e^{2}e^{-j\pi} = -e^{2}(-1) = e^{2}$

$x = e^{2} \qquad y = 0$

C. $\frac{\sqrt{2}}{5}\left(e^{j\pi/4}\right) = \frac{1}{5}(1 + j)$

$x = \frac{1}{5} \qquad y = \frac{1}{5}$

D. $3e^{0j} = 3e^{0} = 3$

$x = 3 \qquad y = 0$

E. $j\left(e^{j7\pi}\right) = je^{j\pi}e^{j6\pi} = je^{j\pi} = -j$

$x = 0 \qquad y = -1$

F. $-e^{j3\pi} = -e^{j\pi}e^{2\pi} = -e^{j\pi} = -(-1) = 1$

$x = 1 \qquad y = 0$

G.

$2e^{\left(1+j\frac{\pi}{2}\right)} = 2e\left(e^{j\frac{\pi}{2}}\right) = 2e(j)$

$x = 0 \qquad y = 2e$

H. $\pi\sqrt{2}\left(e^{-j\pi/4}\right) = \pi\sqrt{2}\left(\frac{1-j}{\sqrt{2}}\right) = \pi(1-j)$

$x = \pi \qquad y = -\pi$

2. (10 pts.) Specify r and θ in the polar form $re^{j\theta}$ for following complex numbers, drawing a picture of each on the complex plane.
($-\pi < \theta \leq \pi$, $r \geq 0$) <--- note range of variables

A. $(1-j)\big/(1+j) = \frac{(1-j)(1-j)}{(1+j)(1-j)} = \frac{1-2j+j^2}{2} = -j$

$\theta = -\frac{\pi}{2} \qquad r = 1$

B. $3(1+j)$

$\theta = \frac{\pi}{4} \qquad r = 3(1+1)^{\frac{1}{2}} = 3\sqrt{2}$

C. $-17 = 17e^{j\pi}$

$\theta = \pi \qquad r = 17$

D. $\sqrt{-4} = 2j$

$\theta = \dfrac{\pi}{2} \qquad r = 2$

E. $-\sqrt{2} - \left(j\sqrt{2}\right)$

$\theta = -\dfrac{3}{4}\pi \qquad r = 2$

F. $-\sqrt{2} = \sqrt{2}\left(e^{\pi}\right)$

$\theta = \pi \qquad r = \sqrt{2}$

G. $\sqrt{-\left(\dfrac{1}{4}\right)} = \dfrac{1}{2}j$

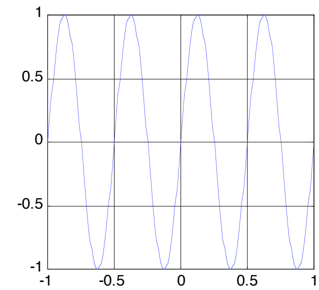
$\theta = \dfrac{\pi}{2} \qquad r = \dfrac{1}{2}$

H. $2j\Big/\left(\sqrt{3} - j\right) = \dfrac{2j\left(\sqrt{3} + j\right)}{\left(\sqrt{3} - j\right)\left(\sqrt{3} + j\right)} = \dfrac{2}{(3+1)}\left(j\sqrt{3} - 1\right) = \dfrac{1}{2}\left(j\sqrt{3} - 1\right)$

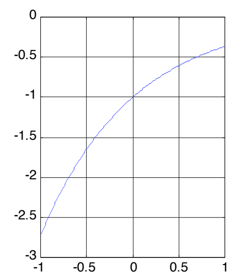
$\theta = \dfrac{2}{3}\pi \qquad r = 1$

3. (15 pts.) For each signal, draw the signal with correct scales for both ordinate and abscissa.

A. $f(t) = \cos(4\pi t - \frac{\pi}{2})$



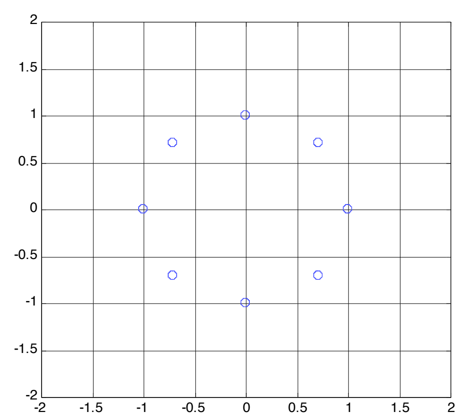
B. $f(t) = -e^{-t}$



4. Given $z = \frac{1}{\sqrt{2}}(1 + j)$

First in Cartesian coordinates, and then using phasors, solve for z^k, $k = 0, 1, 2, 3, \ldots, 8$
Draw a diagram of the nine vectors on the complex plane.

First, convert it to a phasor. $z = \frac{1}{\sqrt{2}}(1 + j) = e^{\frac{\pi}{4}}$. Then raising it to each successive power z^k rotates the unit phasor by another $\frac{\pi}{4} = 45^o$.



5. Using the phasor representations, prove the following trigonometric identity:

$$\sin(2t) = 2\sin(t)\cos(t) =$$
$$2\left[\frac{e^{jt} - e^{-jt}}{2j}\right]\left[\frac{e^{jt} + e^{-jt}}{2}\right] =$$
$$\frac{e^{2jt} - e^{-2jt}}{2j} = \sin(2t)$$

6. Fill in the blanks.

The sine and cosine functions are both examples of a general class of waveform known as **sinusoids**, and differ only in their **phase**. Any two sinusoids with the same **frequency**, when added together, create another sinusoid. Systems obeying Hooke's Law produce sinusoidal motion because the **(negative) second derivative** of a sinusoid equals a constant times that sinusoid. Taking the derivative of a sinusoid shifts the phase by **-90** degrees, which can also be expressed as **-PI/2** radians. The only function whose first derivative equals itself is **e^x** . A phasor (complex exponential) has special properities, including that multiplying one phasor by another shifts the phase of one by the **phase** of the other.

7. True / False (enter "T" or "F" in each blank).

T Every complex number can be represented by a phasor, and visa versa.

T Any sinusoid can be expressed as the sum of two complex exponentials.

F The sum of a complex number and its conjugate can have a non-zero imaginary component.

T The square root of a phasor always has half the angle.