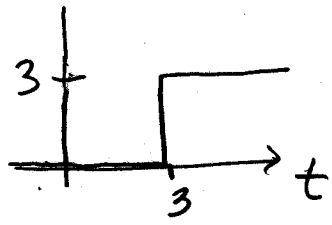
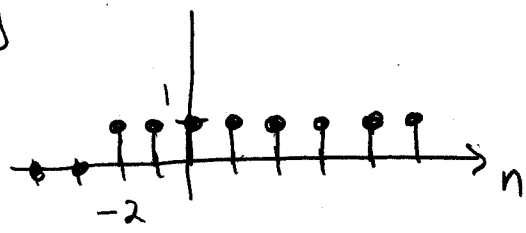
BioE 1410 Homework 2 ANSWERS

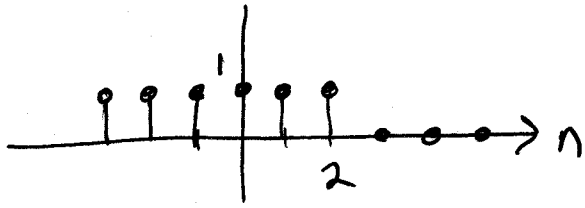
① $3u(t-3)$



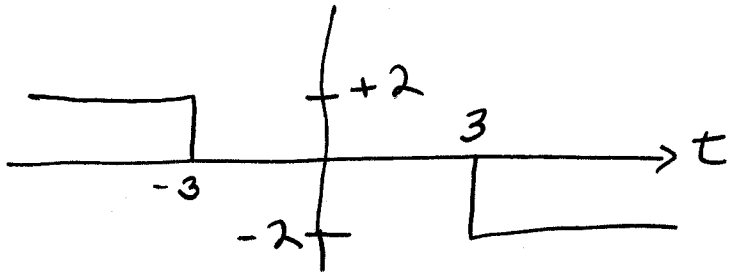
② $u[n+2]$



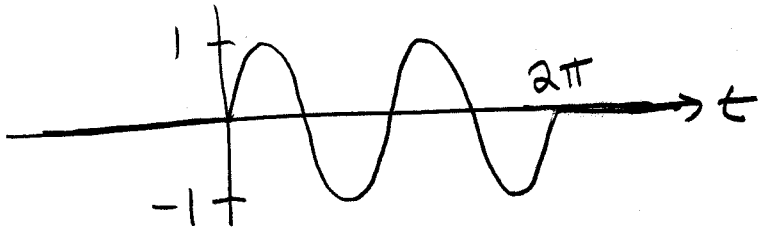
③ $u[2-n]$



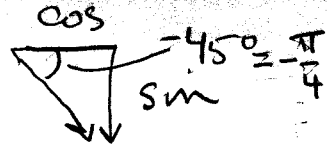
④ $2u(3-t) - 2u(t+3)$

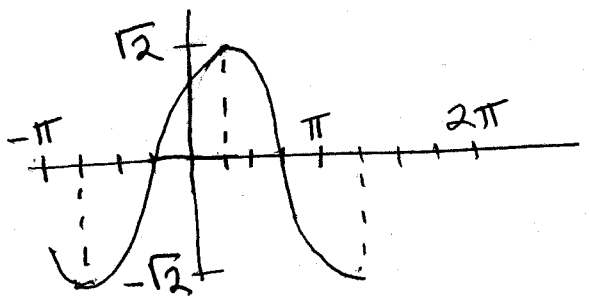


⑤ $[u(t) - u(t-2\pi)]\sin 2t$



⑥ $\cos(t) + \sin(t)$





$$\cos(t) = \frac{1}{2}e^{jt} + \frac{1}{2}e^{-jt}$$

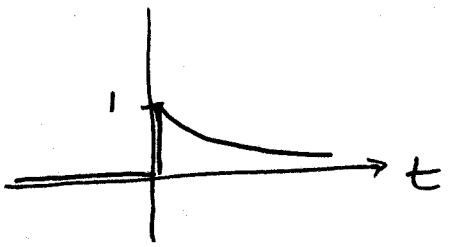
$$\sin(t) = -\frac{j}{2}e^{jt} + \frac{j}{2}e^{jt}$$

$$\cos(t) + \sin(t) = \frac{(1-j)e^{jt} + (1+j)e^{-jt}}{2}$$

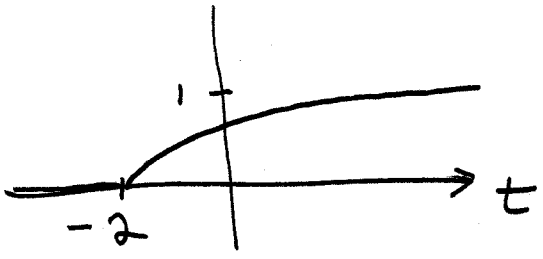
$$= \sqrt{2}\,\frac{e^{j(t-\frac{\pi}{4})} + e^{-j(t-\frac{\pi}{4})}}{2}$$

$$= \sqrt{2}\cos\left(t - \frac{\pi}{4}\right)$$

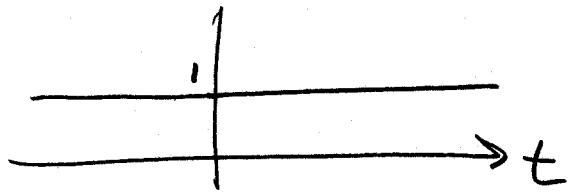
⑦ $u(t)e^{-t}$



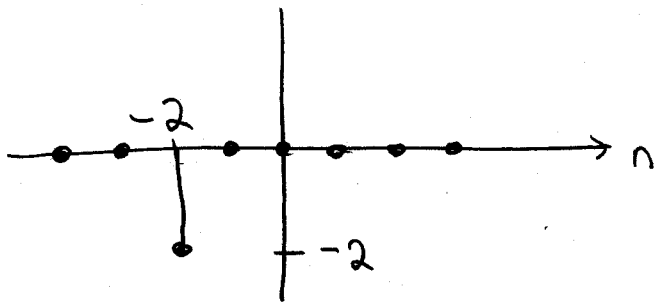
⑧ $\left(1 - e^{-(t+2)}\right)u(t+2)$



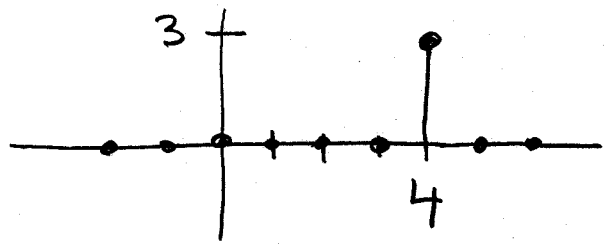
(9) $\sin^2(t) + \cos^2(t) = 1$



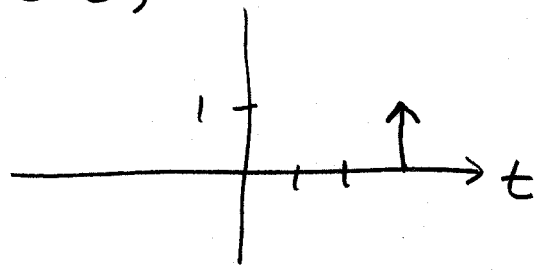
(10) $-2\delta[n+2]$



(11) $3\,\delta[4-n]$



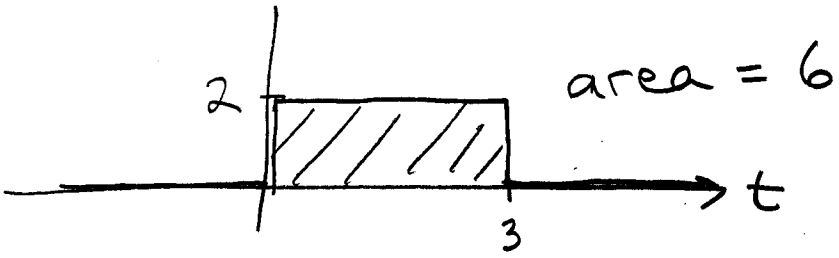
(12) $\delta(t-3)$



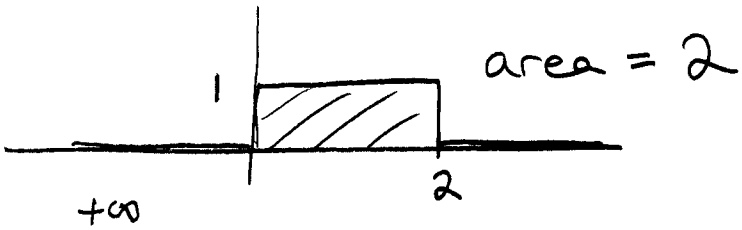
(13) $\int_{-\infty}^{+\infty} t^2 \delta(t-3)\,dt = t^2\Big|_{t=3} = 9$

(14) $$\int_{-\infty}^{+\infty} \cos^2(2t)\,\delta\left(t+\frac{\pi}{2}\right)dt = \cos^2(2t)\Big|_{t=-\frac{\pi}{2}} = 1$$

(15) $2u(t)u(3-t)$



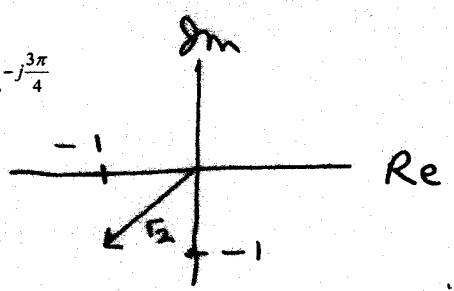
(16) $u(t)-u(t-2)$



$$6\int_{-\infty}^{+\infty}[u(t)-u(t-2)]\,dt = 12$$

17. ~~1. (10 points)~~ Express each of the following complex numbers in Cartesian form $x + jy$, clearly specifying the real and imaginary parts and drawing a picture of each on the complex plane. Do not include any exponentials or trigonometric expressions in your answers.

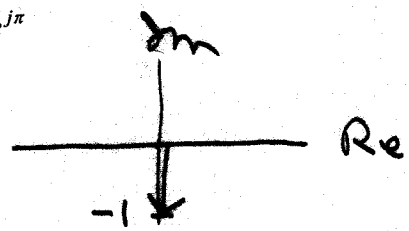
A. $\sqrt{2}e^{-j\frac{3\pi}{4}}$



$-1 - j$

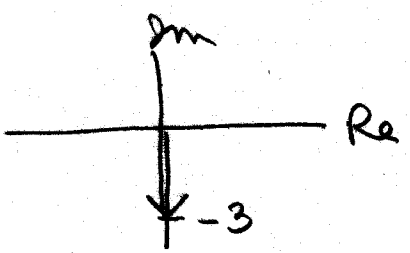
B. $je^{j\pi}$

$e^{j\pi} = -1$



$0 - j$

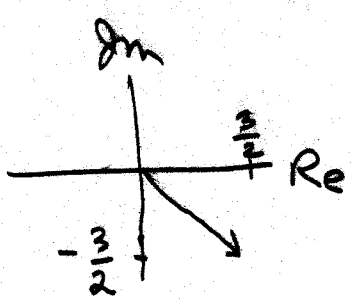
C. $-3e^{j\frac{\pi}{2}}$



$0 - 3j$

18. ~~2. (10 points)~~ Specify r and θ in the polar form $re^{j\theta}$ for following complex numbers, drawing a picture of each on the complex plane.
$(-\pi < \theta \leq \pi,\ r \geq 0)$ <--- note range of variables

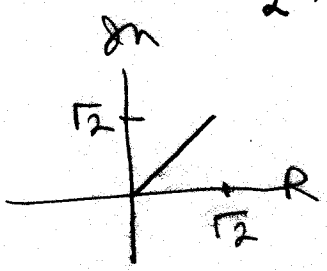
A. $\frac{3}{(1+j)}\frac{1-j}{1-j} = \frac{3-3j}{1-j^2} = \frac{3}{2}(1-j) =$



$\frac{3}{2}\sqrt{2}\,e^{-j\frac{\pi}{4}} = \frac{3}{\sqrt{2}}e^{-j\frac{\pi}{4}}$

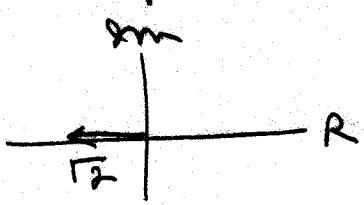
B. $\sqrt{2} + \sqrt{-2} = \sqrt{2} + j\sqrt{2} =$

$2e^{j\frac{\pi}{4}}$



C. $-\sqrt{2}$

$\sqrt{2}\,e^{j\pi}$



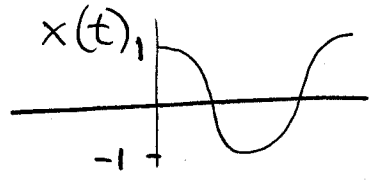
19. ~~3. (10 points)~~ Using the phasor representations, prove the following trigonometric identity:

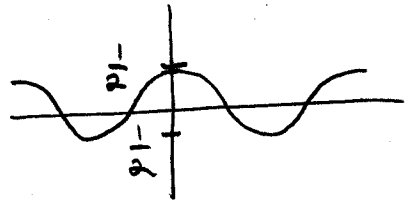
$$\sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha-\beta)+\frac{1}{2}\sin(\alpha+\beta)$$

$$\frac{e^{j\alpha}-e^{-j\alpha}}{2j}\cdot\frac{e^{j\beta}+e^{-j\beta}}{2} = \underbrace{\frac{e^{j(\alpha+\beta)}-e^{-j(\alpha+\beta)}}{4j}}_{\frac{1}{2}\sin(\alpha+\beta)} + \underbrace{\frac{e^{j(\alpha-\beta)}-e^{-j(\alpha-\beta)}}{4j}}_{\frac{1}{2}\sin(\alpha-\beta)}$$

20 ~~4. (10 points)~~ Find the even and odd parts of the following function, and graph each part:

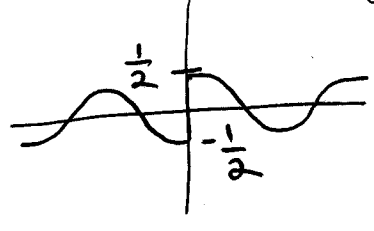
$$x(t) = u(t)\cos(t)$$





$$Ev\{x(t)\} = \frac{1}{2}u(t)\cos(t) + \frac{1}{2}u(-t)\cos(-t) = \frac{1}{2}\cos(t)$$

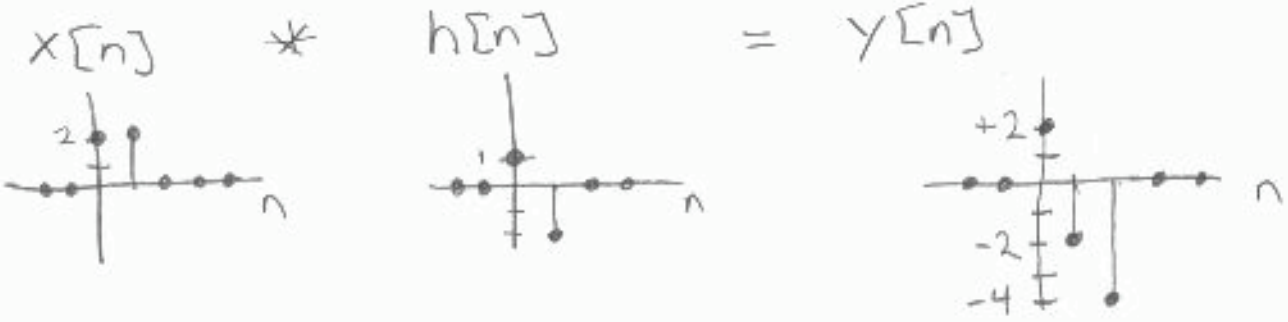
$$Od\{x(t)\} = \frac{1}{2}u(t)\cos(t) - \frac{1}{2}u(-t)\cos(-t)$$



24.

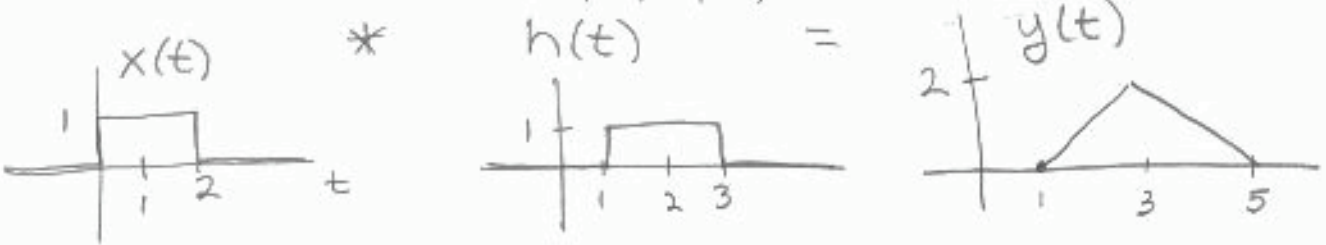
A. Draw a picture of $x[n]$ and $h[n]$ and solve for $y[n] = x[n] * h[n]$ graphically, producing a drawing of $y[n]$. Label axes and all important coordinates.

$x[n] = 2(u[n] - u[n-2])$, $h[n] = \delta[n] - 2\delta[n-1]$



B. Draw a picture of $x(t)$ and $h(t)$ and solve for $y(t) = x(t) * h(t)$ graphically, producing a drawing of $y(t)$. Label axes and all important coordinates.

$x(t) = u(t)u(2-t)$, $h(t) = u(t-1) - u(t-3)$



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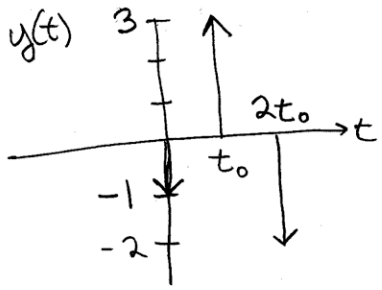

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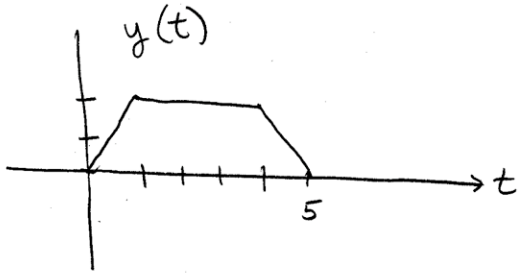

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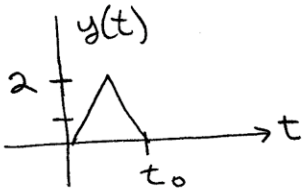

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