Homework 3 Answers

① $a_0 = 3$

$a_1 = \frac{3}{2} + \frac{j}{2}$ $\quad a_{-1} = \frac{3}{2} - \frac{j}{2}$

$a_2 = -\frac{j}{2}$ $\quad a_{-2} = \frac{j}{2}$

$\omega_0 = 2 \leftarrow$ corresponds to the shortest T_0 for which $x(t)$ is periodic

② $\sin(6t + \pi/3) = \dfrac{e^{j(6t+\pi/3)} - e^{-j(6t+\pi/3)}}{2j} =$

$-\frac{j}{2} e^{j\pi/3} e^{j6t} + \frac{j}{2} e^{-j\pi/3} e^{-j6t}$

$a_1 = \dfrac{e^{j(\frac{\pi}{3}-\frac{\pi}{2})}}{2} = \dfrac{e^{-j\frac{\pi}{6}}}{2}$ $\quad a_{-1} = \dfrac{e^{-j(\frac{\pi}{3}-\frac{\pi}{2})}}{2} = \dfrac{e^{j\frac{\pi}{6}}}{2}$

$a_1 = \frac{\sqrt{3}}{4} - \frac{j}{4}$ $\quad a_{-1} = \frac{\sqrt{3}}{4} + \frac{j}{4}$

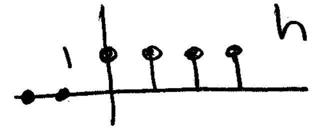
$a_0 = -1$

③ $x(t) = 2 + 2\cos(\omega_0 t) - 2\sin(\omega_0 t)$

④ The phasor $e^{-jk\omega_0 t}$ spins $x(t)$ backwards so that the k^{th} harmonic stands still to be integrated while all the other harmonics integrate to zero because they are spinning.

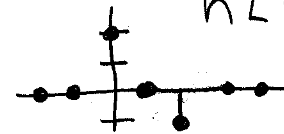
$a_0 = \frac{1}{T_0}\int_{T_0} x(t)e^0\,dt = \frac{1}{T_0}\int_{T_0} x(t)\,dt$

(5) memory, causal, IIR



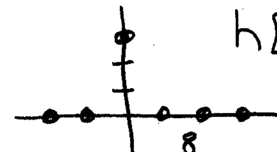
$h[n] = u[n]$

(6) memory, causal, FIR



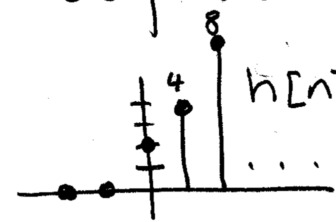
$h[n] = 2\delta[n] - \delta[n-2]$

(7) no memory, causal, FIR



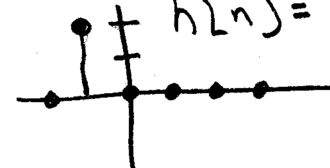
$h[n] = 3\delta[n]$

(8) memory, causal, IIR



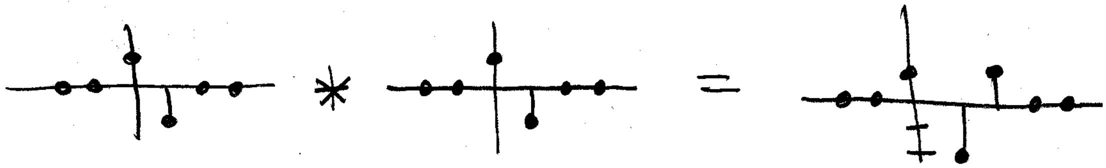
$h[n] = 2^{n+1} u[n]$

(9) memory, not causal, FIR



$h[n] = \delta[n+1]$

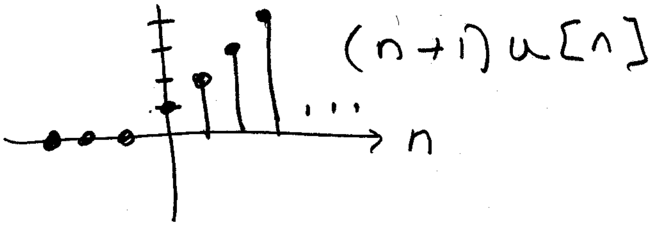
(10)



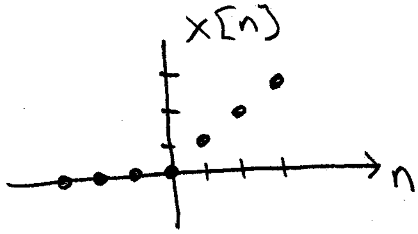
(11) 2 first derivatives = a second derivative

(12) 2 integrals $\rightarrow \boxed{u[n]} \rightarrow \boxed{u[n]} \rightarrow$

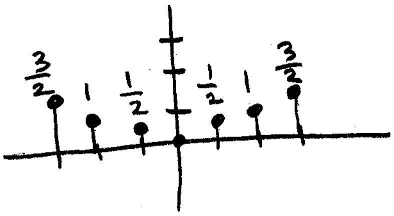
(13)



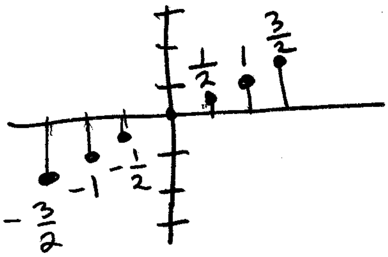
(14)



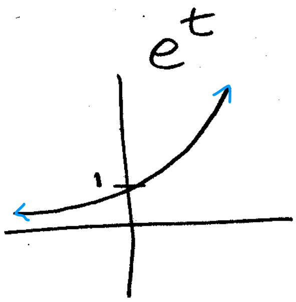
$$Ev\{x[n]\} = \frac{1}{2}\left(u[n]\cdot n + u[-n]\cdot(-n)\right)$$



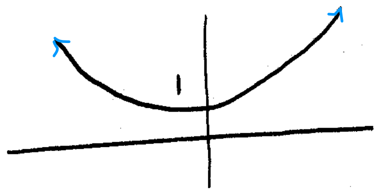
$$Od\{x[n]\} = \frac{1}{2}\left(u[n]\cdot n - u[-n]\cdot(-n)\right)$$



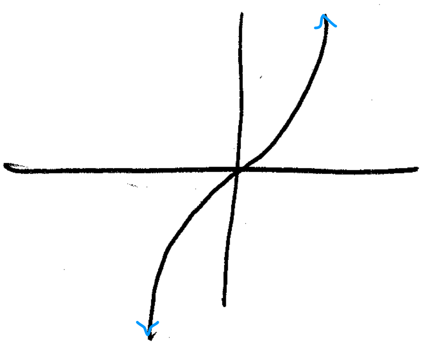
(15)



$$Ev\{e^t\} = \frac{1}{2}\left[e^t + e^{-t}\right]$$



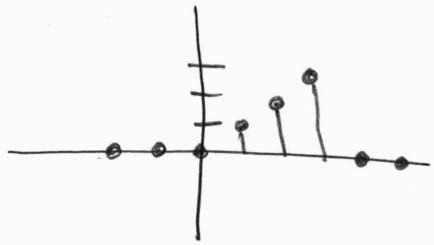
$$Od\{e^t\} = \frac{1}{2}\left[e^t - e^{-t}\right]$$



16.

A

$$y[n] = x[n-1] + 2x[n-2] + 3x[n-3]$$

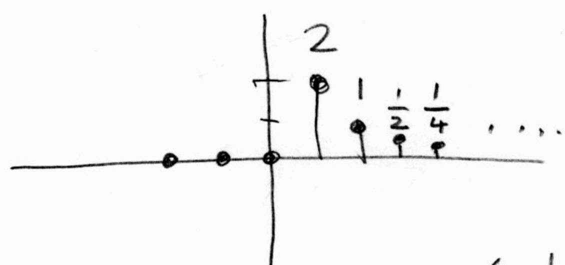


$$h[n] = \delta[n-1] + 2\delta[n-2] + 3\delta[n-3]$$

FIR

B

$$y[n] = 2x[n-1] + \frac{1}{2}y[n-1]$$



$$h[n] = 2u[n-1]\left(\frac{1}{2}\right)^{n-1}$$

IIR