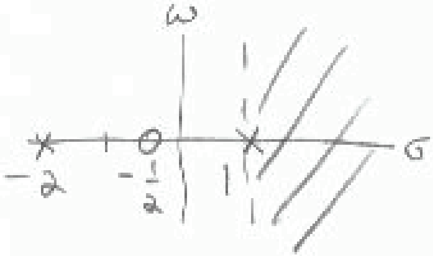
Homework 6 answers

1

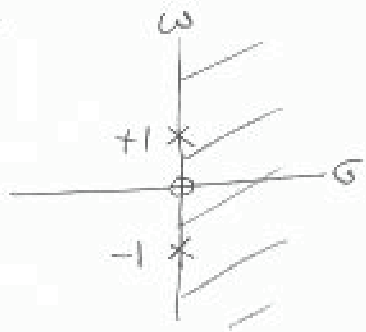
A) $X(s) = -2$, no poles or zeros, stable

B) $X(s) = \frac{1}{s-1} + \frac{1}{s+2} = \frac{2(s+\frac{1}{2})}{(s-1)(s+2)}$



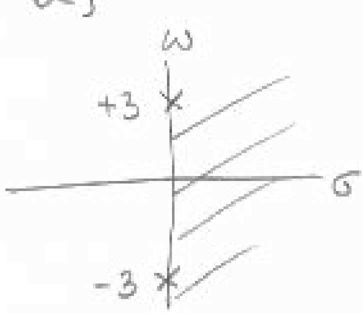
unstable

C) $\frac{e^{jt}+e^{-jt}}{2}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{2}\left[\frac{1}{s-j}+\frac{1}{s+j}\right] = \frac{s}{(s-j)(s+j)}$



unstable

D) $-2\frac{e^{j3t}-e^{-j3t}}{2j}u(t) \overset{\mathcal{L}}{\longleftrightarrow} -2\left[\frac{1}{s-3j} - \frac{1}{s+3j}\right] = \frac{-6}{(s-3j)(s+3j)}$

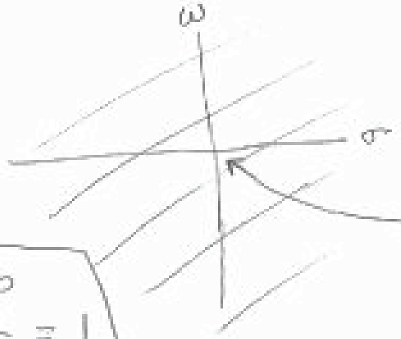


unstable

E) $X(s) = \dfrac{1-e^{-s}}{s}$

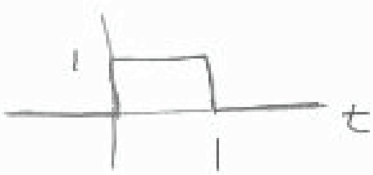
both the numerator and denominator have roots at $s=0$,

l'hopital's rule says $X(s)\big|_0 = \dfrac{\frac{d(1-e^{-s})}{ds}}{\frac{ds}{ds}}\Bigg|_0 = \dfrac{e^0}{1} = 1$



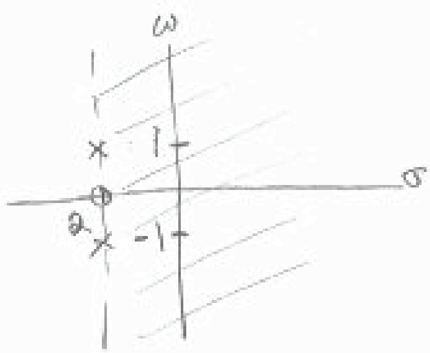
pole and zero at cancel each other

looking at $x(t)$, a system with this impulse response would clearly be **stable**



F) $e^{-2t}\dfrac{e^{jt}+e^{-jt}}{2}u(t) = \dfrac{e^{-(2-j)t}+e^{-(2+j)t}}{2}u(t)$

$X(s) = \dfrac{1}{2}\left[\dfrac{1}{s+2-j}+\dfrac{1}{s+2+j}\right] = \dfrac{(s+2)}{(s+2-j)(s+2+j)}$



Stable

(2) $H(s) = \frac{1}{s^2+1} = \frac{1}{(s+j)(s-j)} =$

$$\frac{c_1}{(s+j)} + \frac{c_2}{(s-j)}$$

$$c_1 = \left[X(s)(s+j)\right]_{s=-j} = -\frac{1}{2j}$$

$$c_2 = \left[X(s)(s-j)\right]_{s=j} = \frac{1}{2j}$$

$$h(t) = -\frac{1}{2j}e^{-jt} + \frac{1}{2j}e^{jt} = \sin(t)u(t)$$

unstable

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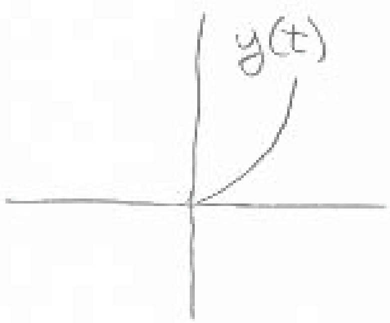
(3) $x(t) = e^t u(t)$

$y(t) = 0$ for $t < 0$

For $t \geq 0$

$$y(t) = \int_0^t e^{\tau} d\tau = e^t - 1$$



So

$$y(t) = e^t u(t) - u(t)$$

$X(s) = \frac{1}{s-1}$ $\qquad Y(s) = \frac{1}{s-1} - \frac{1}{s}$

$H(s) = \frac{1}{s}$

$X(s) \cdot H(s) \stackrel{?}{=} Y(s)$

$$\frac{1}{s-1} \cdot \frac{1}{s} \stackrel{?}{=} \frac{1}{s-1} - \frac{1}{s} = \frac{s-s+1}{(s-1)s} = \frac{1}{(s-1)s}$$

QED

(4) $$\frac{dy(t)}{dt} + \frac{1}{\tau} y(t) = b x(t)$$

assume causality,

$$sY(s) + \frac{1}{\tau} Y(s) = b X(s)$$

(A) (B)

$$H(s) = \frac{Y(s)}{X(s)} = \frac{b}{s + \frac{1}{\tau}} \overset{\mathcal{L}}{\longleftrightarrow} b e^{-\frac{1}{\tau} t} u(t)$$

$$x(t) = u(t) = e^{-0t} u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s} = X(s) \quad \text{(C)}$$

$$Y(s) = H(s)\frac{1}{s} = \left(\frac{b}{s + \frac{1}{\tau}}\right)\frac{1}{s} = \frac{c_1}{s} + \frac{c_2}{s + \frac{1}{\tau}}$$

$$c_1 = \left[s Y(s) \right]\Big|_{s=0} = \frac{b}{s + \frac{1}{\tau}}\Big|_{s=0} = b\tau$$

$$c_2 \left[\left(s + \frac{1}{\tau}\right) Y_s \right]\Big|_{s=-\frac{1}{\tau}} = \frac{b}{s}\Big|_{s=-\frac{1}{\tau}} = -b\tau$$

(D) $$Y(s) = \frac{b\tau}{s} - \frac{b\tau}{s + \frac{1}{\tau}}$$

(E)

$$y(t) = b\tau - b\tau e^{-\frac{t}{\tau}} = b\tau\left(1 - e^{-t/\tau}\right) u(t)$$