HOMEWORK 7 - Answers

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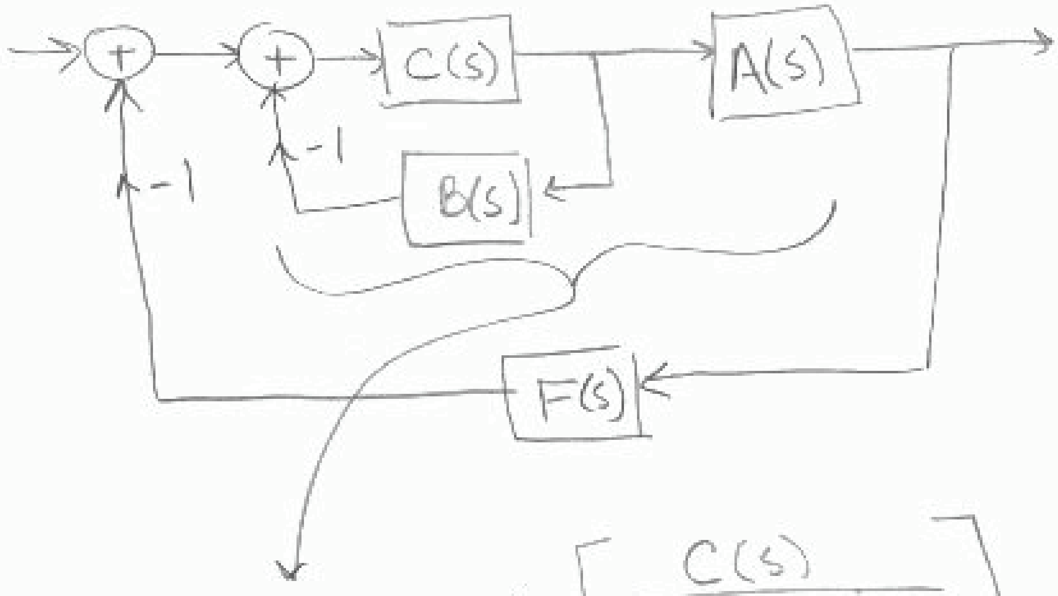


open loop gain $$G(s) = A(s)\left[\frac{C(s)}{1+B(s)C(s)}\right]$$

$$H(s) = \frac{G(s)}{1+F(s)G(s)} = \frac{A(s)\,C(s)}{1+C(s)\left[B(s)+F(s)A(s)\right]}$$

see Bruce for math

given $$A(s) = \frac{2}{s+10}$$

$$B(s) = 3s$$

$$C(s) = \frac{s+5}{s+10}$$

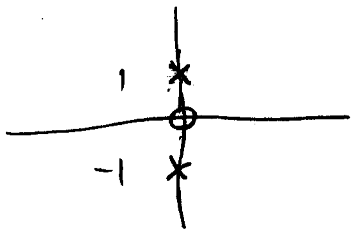
$$F(s) = \frac{1}{s}$$

$$H(s) = \frac{2s^2+10s}{3s^4+46s^3+170s^2+102s+10}$$

$$3\frac{dy^4(t)}{dt^4} + 46\frac{dy^3(t)}{dt^3} + 170\frac{dy^2(t)}{dt^2} + 102\frac{dy(t)}{dt} + 10y(t) = 2\frac{d^2x(t)}{dt^2} + 10\frac{dx(t)}{dt}$$

(2) $\cos(t)\,u(t) = \dfrac{e^{jt} + e^{-jt}}{2}\,u(t)$

$$G(s) = \frac{1}{2}\left[\frac{1}{s-j} + \frac{1}{s+j}\right] = \frac{s}{(s-j)(s+j)}$$



$$H(s) = \frac{G(s)}{1 + F(s)G(s)} = \frac{s}{(s-j)(s+j) + sF(s)}$$

$F(s) = K$

$$H(s) = \frac{s}{\underbrace{s^2 + Ks + 1}}$$

when $k = 2$, denominator is

$$s^2 + 2s + 1 = (s+1)(s+1)$$

both poles are at -1

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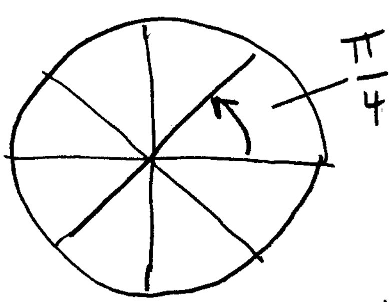
(3)



motion by $n\frac{\pi}{4}$ in $\frac{1}{24}$ sec (when $n = 0, \pm 1, \pm 2 \ldots$) will be undetectable. Corresponds to $3n$ cps. So any velocity of $1 + 3n$ cps will appear to be 1 cps

④ A)

1rst harmonic spinning backwards

average value

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \underbrace{\frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}}_{F^{-1}} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \Rightarrow \left.\begin{matrix} a_0 = 0 \\ a_1 = \frac{1}{2} \\ a_2 = 0 \\ a_3 = \frac{1}{2} \end{matrix}\right\} \text{F.S. of } \cos\left[\tfrac{\pi}{2}n\right]$$

$N = 4$

we can recreate $x[n]$ by multiplying by F

$$\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix} \leftarrow a_k$$

1rst harmonic

B) $N = 3$

average value

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \frac{1}{3}\overbrace{\begin{bmatrix} e^0 & e^0 & e^0 \\ e^0 & e^{-j\frac{2}{3}\pi} & e^{-j\frac{4}{3}\pi} \\ e^0 & e^{-j\frac{4}{3}\pi} & e^{-j\frac{8}{3}\pi} \end{bmatrix}}^{F^{-1}} \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$$

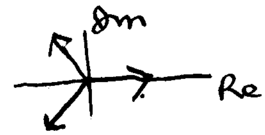
$a_0 = 1$

$a_1 = e^{-j\frac{2}{3}\pi}$

$a_2 = e^{-j\frac{4}{3}\pi}$

$$\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} e^0 & e^0 & e^0 \\ e^0 & e^{j\frac{2}{3}\pi} & e^{j\frac{4}{3}\pi} \\ e^0 & e^{j\frac{4}{3}\pi} & e^{j\frac{8}{3}\pi} \end{bmatrix} \begin{bmatrix} 1 \\ e^{-j\frac{2}{3}\pi} \\ e^{-j\frac{4}{3}\pi} \end{bmatrix}$$

each of these are the sum of 3 evenly-spaced angular unit vectors

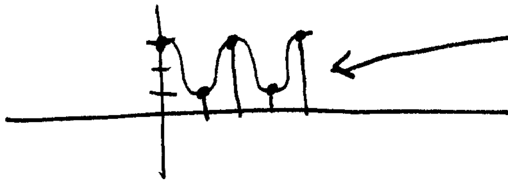


C) $N = 2$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \overbrace{\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}^{F^{-1}} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$a_0 = 2$ ← average value

$a_1 = 1$ cosine



$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}_{F} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

D) $N = 1$ F^{-1} is a 1×1 matrix

$$[a_0] = \overbrace{1\,[1]}^{F^{-1}}\,[5]$$

$a_0 = 5$

$$[5] = \overbrace{[1]}^{F}\,[5]$$

$x[0]$, and all other $x[n]$ as well

E) $N=3$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \frac{1}{3} \underbrace{\begin{bmatrix} e^0 & e^0 & e^0 \\ e^0 & e^{-j\frac{2}{3}\pi} & e^{-j\frac{4}{3}\pi} \\ e^0 & e^{-j\frac{4}{3}\pi} & e^{-j\frac{8}{3}\pi} \end{bmatrix}}_{F^{-1}\text{, same as in B)}} \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$a_0 = 1$

$a_1 = 1$

$a_2 = 1$

$$\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} e^0 & e^0 & e^0 \\ e^0 & e^{j\frac{2}{3}\pi} & e^{j\frac{4}{3}\pi} \\ e^0 & e^{j\frac{4}{3}\pi} & e^{j\frac{8}{3}\pi} \end{bmatrix}}_{F\text{, same as in B)}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$x[n-n_0] \longleftrightarrow a_k e^{-jk\omega_0 n}$$

$$n_0 = -1, \quad \omega_0 = \frac{2}{3}\pi$$