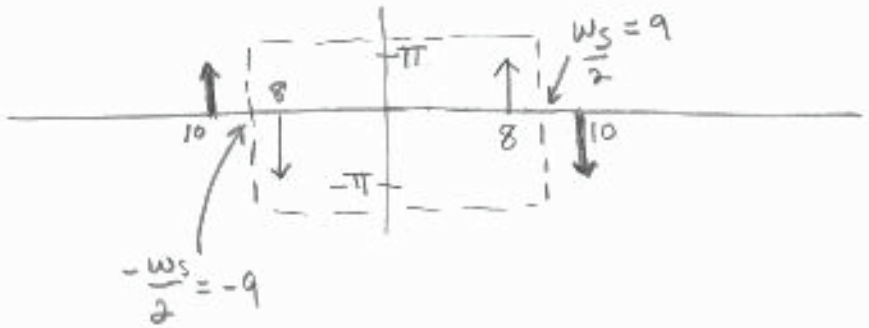
Homework 8 answers

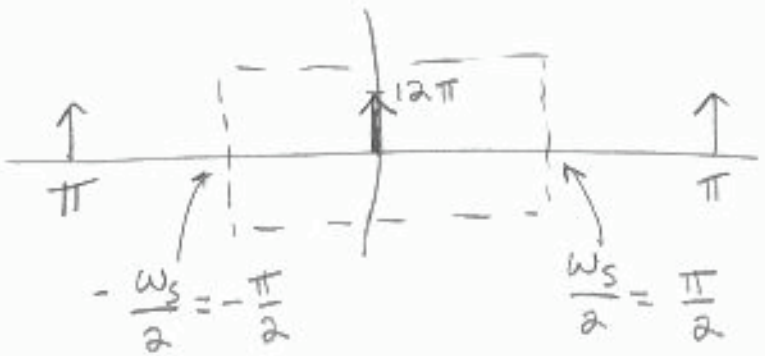
① A) $\mathrm{Im}\{X(\omega)\}$



$\frac{\omega_s}{2} = 9$

yes, aliased signal is $-\sin(8t)$

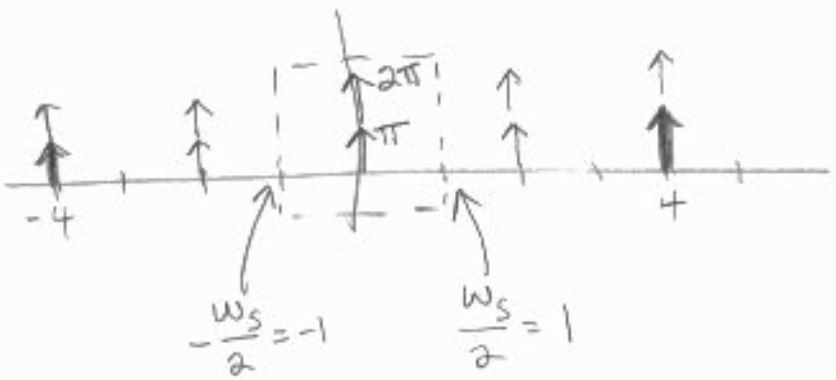
B) $\mathrm{Re}\{X(\omega)\}$



no aliasing,

$\mathrm{Re}\{X(\omega)\}$

C)



yes aliasedsignal is a constant 1

(2) Find the Z transform of

$$x[n] = 3\delta[n] + \delta[n-2] + \delta[n+2]$$

since $x[n-n_0] \overset{Z}{\Longleftrightarrow} z^{-n_0} X(z)$

$$X(z) = 3 + z^{-2} + z^{2}$$

since $x[n]$ is finite length (stable)

ROC is entire Z plane.

no poles

(4) $x[n] = \cos[n\omega_0]u[n]$

$$= \frac{e^{jn\omega_0} + e^{-jn\omega_0}}{2} u[n]$$

$a = e^{j\omega_0}$ $\qquad$ $a = e^{-j\omega_0}$

recall that

$$a^n u[n] \xleftrightarrow{z} \frac{z}{z-a} = \frac{1}{1-az^{-1}}$$

$$X(z) = \frac{1}{2}\left(\frac{1}{1-e^{j\omega_0}z^{-1}} + \frac{1}{1-e^{-j\omega_0}z^{-1}} \right) \leftarrow \text{poles at } z=1$$

$$= \frac{1 - \left[\frac{e^{j\omega_0}+e^{-j\omega_0}}{2}\right] z^{-1}}{1 - \left[e^{j\omega_0}+e^{-j\omega_0}\right] z^{-1} + z^{-2}}$$

$$= \boxed{\frac{1-(\cos\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}}$$

(5) $X(z) = \sum_{n=-\infty}^{+\infty} x[n] z^{-n}$

$m = -n$

$x[-n] \xleftrightarrow{z} \sum_{m=-\infty}^{+\infty} x[m] z^{-m} =$

$\sum_{n=-\infty}^{+\infty} x[n] z^{n} = X\left(\frac{1}{z}\right)$