"SIFTING"

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k] \overbrace{\delta[n-k]}^{\text{"fires" when } n=k}$$

$$x(t) = \int_{-\infty}^{+\infty} x(\tau) \overbrace{\delta(t-\tau)}^{\text{"fires" when } t=\tau} d\tau$$

The scaled and shifted impulse

$$a\delta[n-k] \quad \text{or} \quad a\delta[t-\tau]$$

provides a "snapshot" or sample of a function, from which the function can be reconstructed. The samples are independend, thus forming an orthogonal basis, just as $re^{j\omega t}$ does, or

$$a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 \ldots\ldots$$

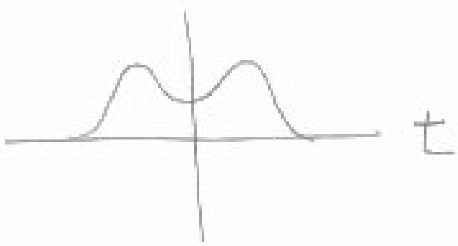
Other orthogonal components (besides $a\delta(t-\tau)$)

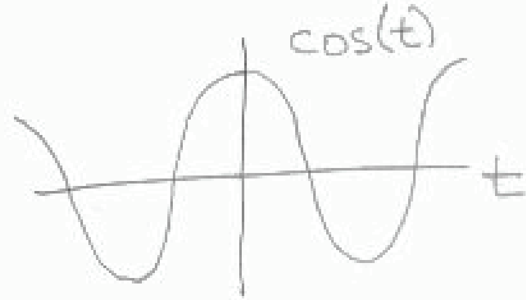
must: ① be independent

② permit the building of any signal

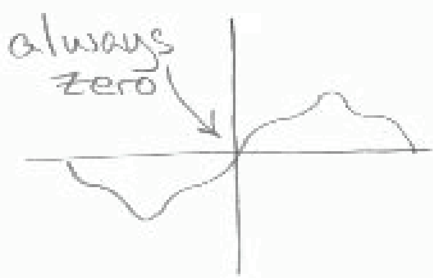
e.g. Even and Odd components

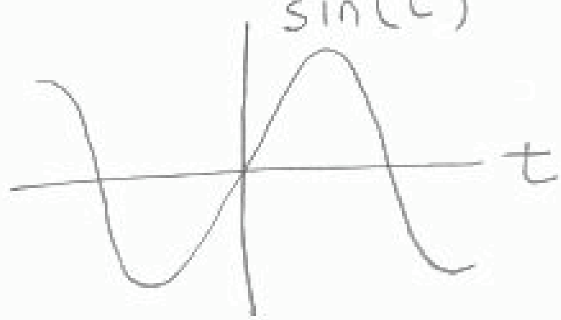
EVEN SIGNAL $\quad x(t) = x(-t)$





ODD SIGNAL $\quad x(t) = -x(-t)$





any $x(t)$ can be broken down into

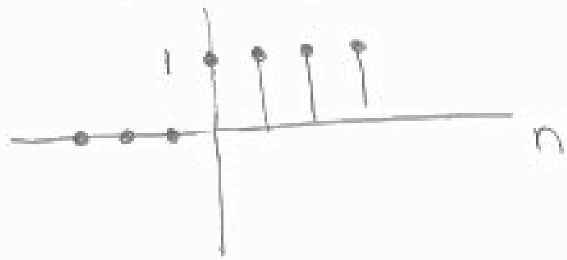
$$\mathcal{E}v\{x(t)\} + \mathcal{O}d\{x(t)\} = x(t)$$

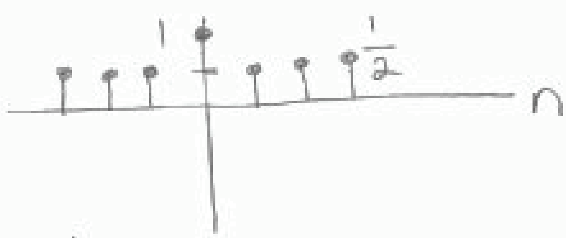
where

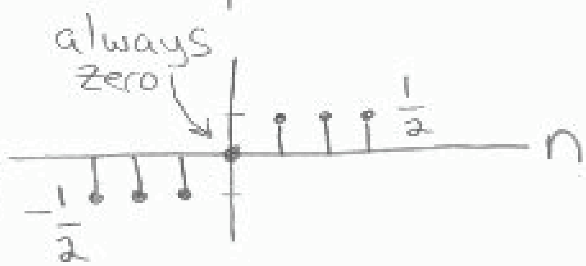
$$\mathcal{E}v\{x(t)\} = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$$\mathcal{O}d\{x(t)\} = \frac{1}{2}\left[x(t) - x(-t)\right]$$

example $x[n] = u[n]$

$x[n]$ 

$\mathcal{E}v\{x[n]\}$ 

$\mathcal{O}d\{x[n]\}$ 

adding even functions together $\Rightarrow$ even function
" odd " " $\Rightarrow$ odd "

Systems

DISCRETE

$$x[n] \rightarrow \boxed{\text{system}} \rightarrow y[n]$$

$y[n]$ can be affected by $x[k]$ or $y[k]$ for any value of k

CONTINUOUS

$$x(t) \rightarrow \boxed{\text{system}} \rightarrow y(t)$$

$y(t)$ can be affected by $x(\tau)$ or $y(\tau)$ for any value of τ

The systems we will study are

$$\underset{\text{linear}}{L}\ \underbrace{T\ I}_{\text{time invariant}}$$

Linear Systems

$$x_1(t) \rightarrow \boxed{} \rightarrow y_1(t)$$

$$x_2(t) \rightarrow \boxed{} \rightarrow y_2(t)$$

$$x_1(t) + x_2(t) \rightarrow \boxed{} \rightarrow y_1(t) + y_2(t)$$

$$A\,x_1(t) \rightarrow \boxed{} \rightarrow A\,y_1(t)$$

$$0 \rightarrow \boxed{} \rightarrow 0$$

note, this is <u>not</u> "linear" as in

$y = mx + b$, ("linear equation")

If a system has

$$0 \rightarrow \boxed{} \rightarrow b$$

it is called

"incremental linear"

not truly linear.

<u>time</u> <u>invariant</u> <u>systems</u>

$$x[n-k] \rightarrow \boxed{\text{system}} \rightarrow y[n-k]$$

$$x(t-\tau) \rightarrow \boxed{\text{system}} \rightarrow y(t-\tau)$$

impulse response

$$\delta[n] \longrightarrow \boxed{\text{system}} \longrightarrow h[n]$$

hitting the bell

clapping your hands in the cathedral

$h[n]$ defines the LTI system

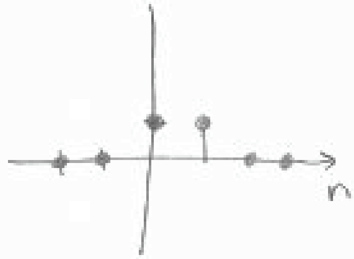
Signals consist of scaled and shifted impulse functions.

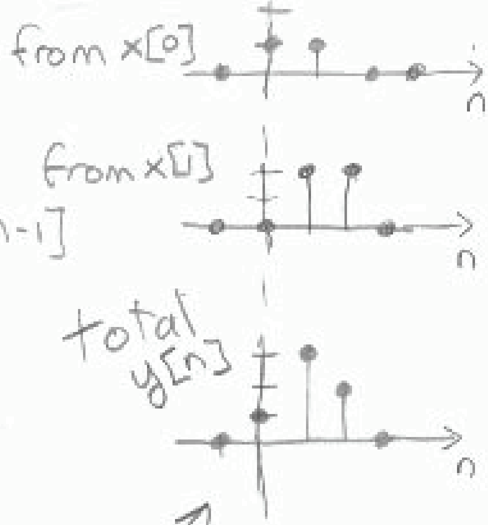
The response of an LTI system is the sum of scaled and shifted impulse responses.

Example of discrete convolution

$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$

$x[n] = \delta[n] + 2\delta[n-1]$ $\qquad$ $h[n] = \delta[n] + \delta[n-1]$





superposition of 2 scaled and shifted impulse responses

Ask the question from the output's point of view... For a given n, what is $y[n]$

$$\boxed{y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]}$$

definition of convolution

why have we flipped $h[k]$ around?

$$y[n] = x[n] * h[n]$$

convolution operator

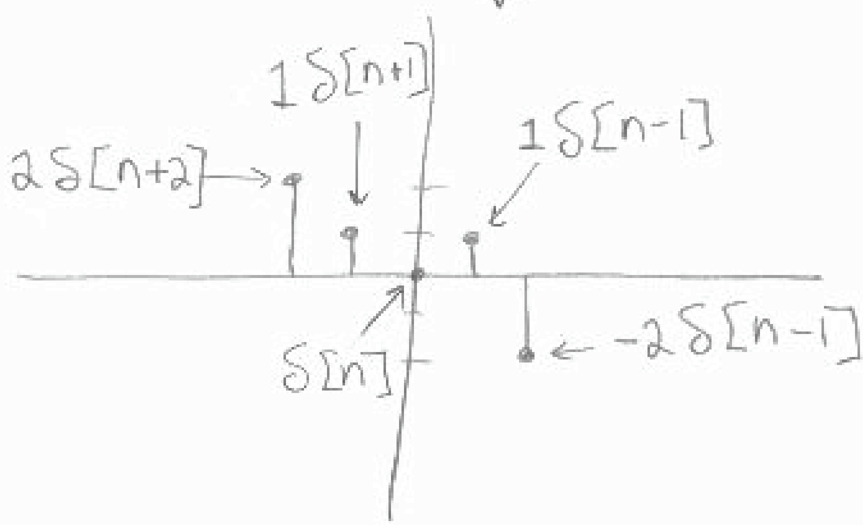
Earlier parts of the input convolve with later parts of the impulse response

This is just like:

$$y[n] = \sum_{k=-\infty}^{+\infty} a_k h[n-k]$$ on the previous page,

with a_k replaced by $x[k]$, which after all makes sense. a_k is the value of $x[n]$ at $n=k$

$x[n]$ consists of impulses

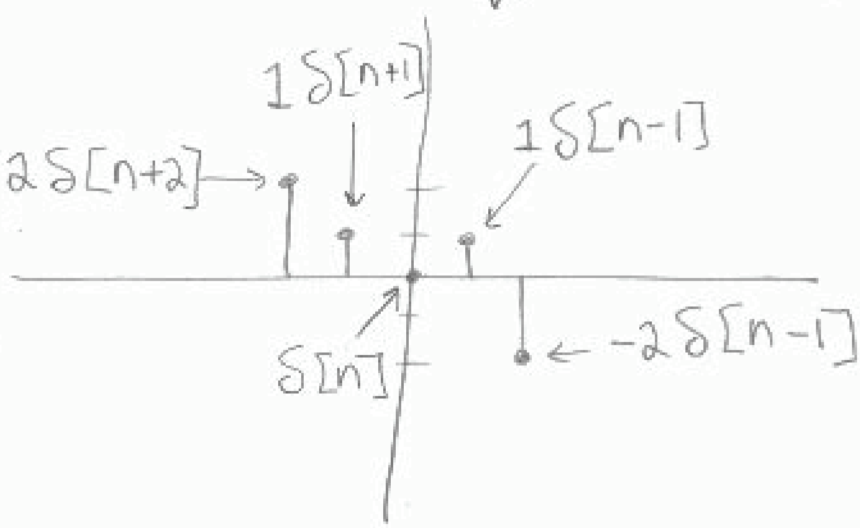


ANY $x[n]$ can be broken up into scaled and shifted impulse functions. These impulse functions can be added together to recreate $x[n]$

$$x[n] = \sum_{k=-\infty}^{+\infty} a_k \delta[n-k] \rightarrow \boxed{\begin{matrix}\text{system}\\ h[n]\end{matrix}} \rightarrow y[n] = \sum_{k=-\infty}^{+\infty} a_k h[n-k]$$

Putting $x[n]$ through the system with impulse response $h[n]$ is the same as putting each impulse through separately and adding afterwards, yielding the sum of a bunch of scaled and shifted impulse responses.

x[n] consists of impulses



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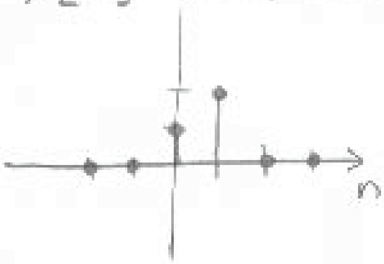
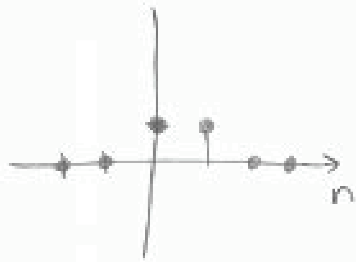
$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$

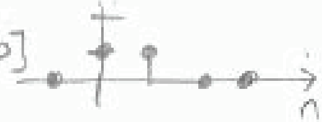
$x[n] = \delta[n] + 2\delta[n-1]$



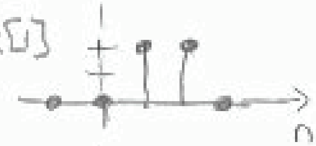
$h[n] = \delta[n] + \delta[n-1]$



from $x[0]$



from $x[1]$



total $y[n]$

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