Continuous system (LTI)

$$\delta(t) \rightarrow \boxed{h(t)} \rightarrow h(t)$$

impulse response

Convolution

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$y(t) = x(t) * h(t)$$

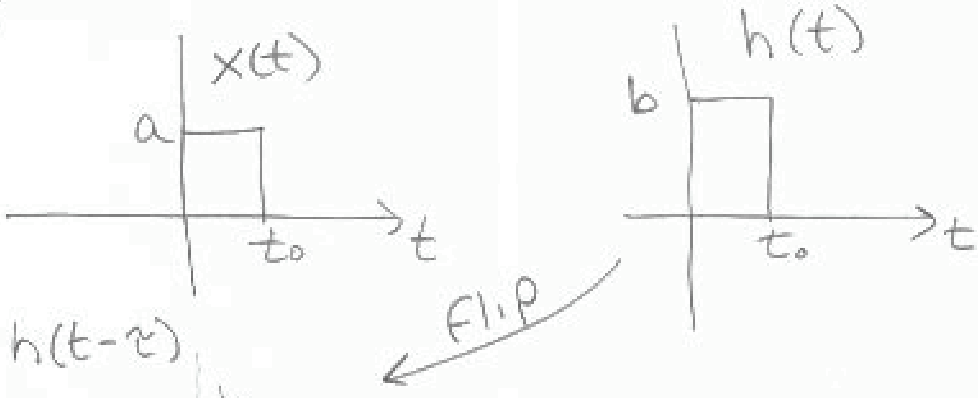
See OPPENHEIM SECTION 3.3

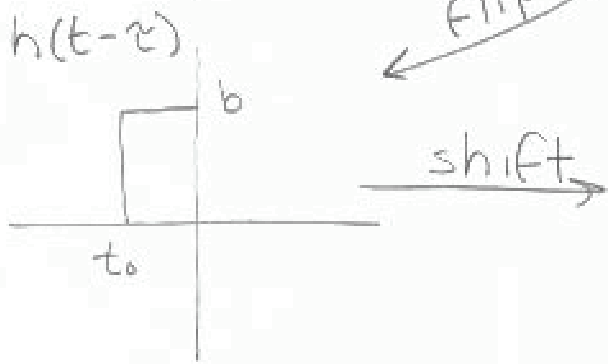
since $x(t) = \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\, d\tau$

For each τ, $\delta(t-\tau)$ goes through the system producing $h(t-\tau)$. These are added to produce $y(t)$

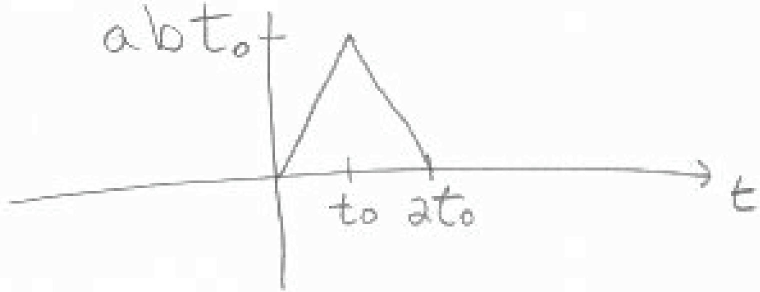
$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau = x(t) * h(t)$$

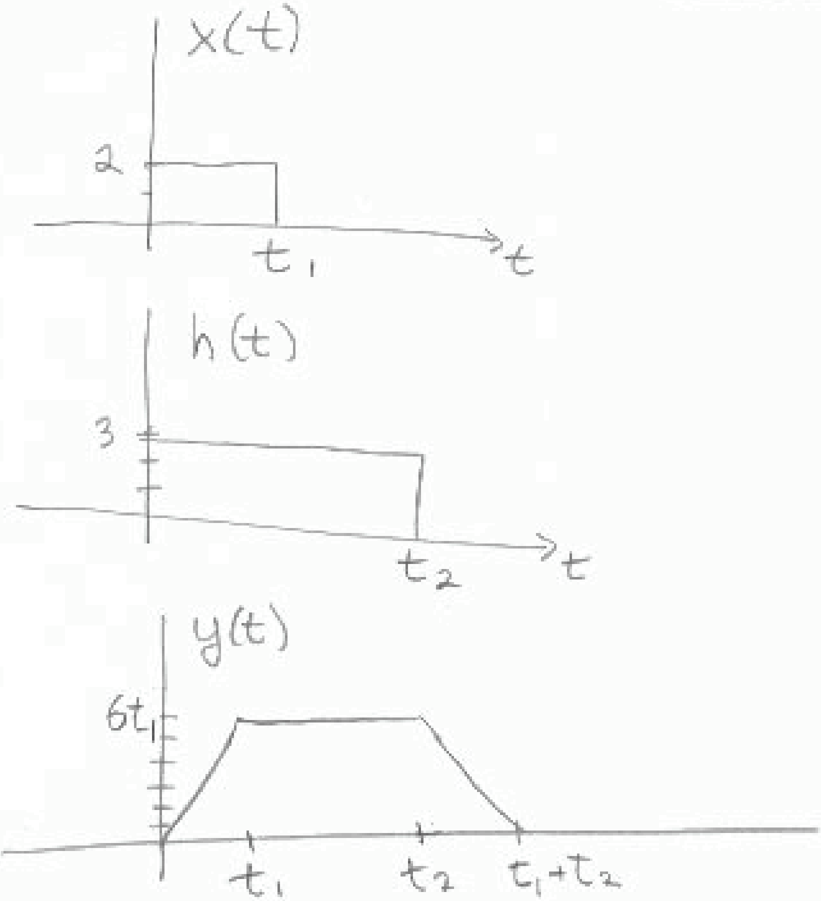
Graphical Convolution





$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$





4 steps to convolution

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t - \tau)\, d\tau$$

$$④ \quad ③ \quad ② \quad ①$$

① <u>Flip</u> - Allows earlier parts of the input $x(\tau)$ to be aligned with later parts in the impulse response $h(\tau)$

② <u>Shift</u> - slides the impulse response $h(\tau)$ to the appropriate t to calculate $y(t)$

③ <u>multiply</u> - the two functions $x(\tau)$ and $h(t-\tau)$, at each τ associating each component impulse in $x(\tau)$ with the particular point in its impulse response, and scaling that impulse response.

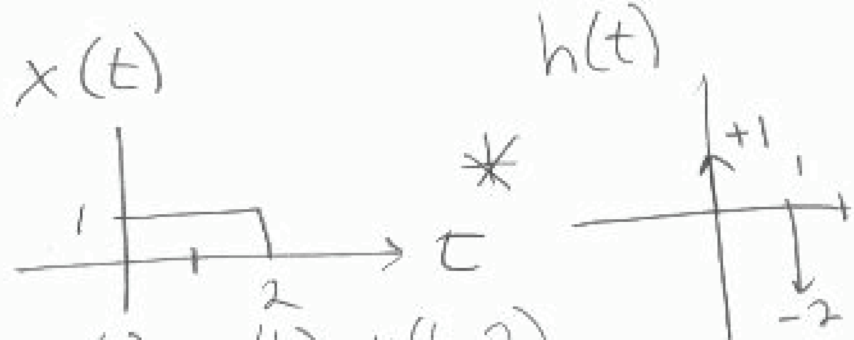
④ <u>Integrate</u> - add all the scaled and shifted impulse responses to compute $y(t)$

$h(t) = \delta(t) - 2\delta(t-1)$

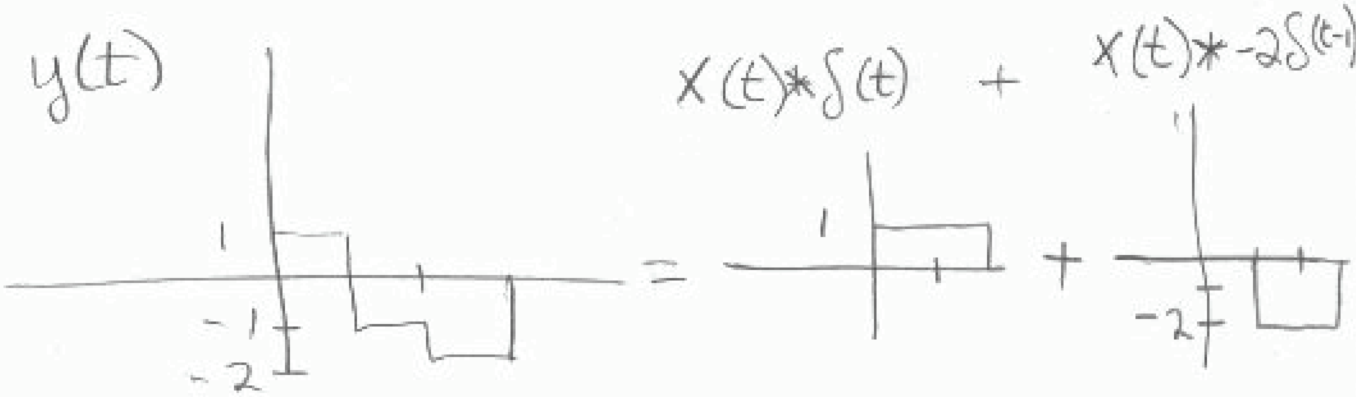
$\delta(t)$: wire

$-2\delta(t-1)$: loud echo

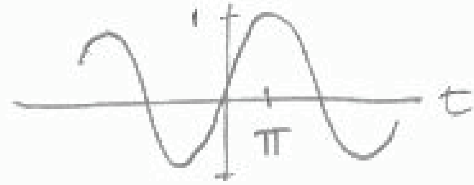
$x(t) = u(t) - u(t-2)$



$x(t) = u(t) - u(t-2)$



$h(t) = \delta(t-\pi)$ $\qquad\qquad$ $x(t) = \sin(t)$



$$y(t) = \int_{-\infty}^{+\infty} \sin(\tau)\, \underbrace{\delta(t-\pi-\tau)}_{h(t-\tau)}\, d\tau$$

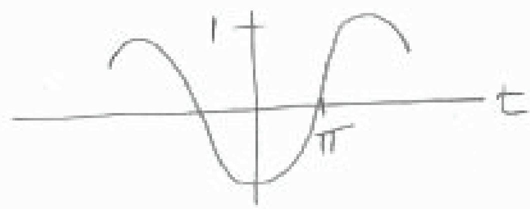
sifting – "flashbulb" takes a picture of $\sin(\tau)$ when

$$(t-\pi-\tau) = 0$$

$$\tau = t - \pi$$

$$y(t) = \sin(t-\pi) = -\sin(\tau)$$

delayed, shifted to the right just like $h(t)$



LTI Systems are <u>Commutative</u>

$$x[n] * h[n] = h[n] * x[n]$$

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$h[n] \longrightarrow \boxed{x[n]} \longrightarrow y[n]$$

system <u>is</u> a signal

signal <u>is</u> a system

proof

$$x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]$$

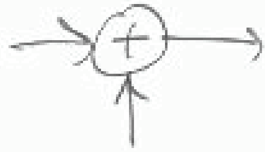
substitute $\boxed{k = n - r}$

$$\sum_{r=+\infty}^{-\infty} x[n-r]\,h[r] = \sum_{r=-\infty}^{+\infty} h[r]\,x[n-r] =$$

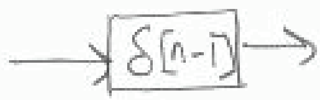
$$h[n] * x[n]$$

DISCRETE SYSTEM DIAGRAMS

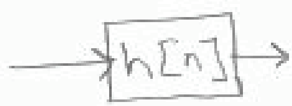
multiply by a, $h[n] = a\delta[n]$



add 2 signals



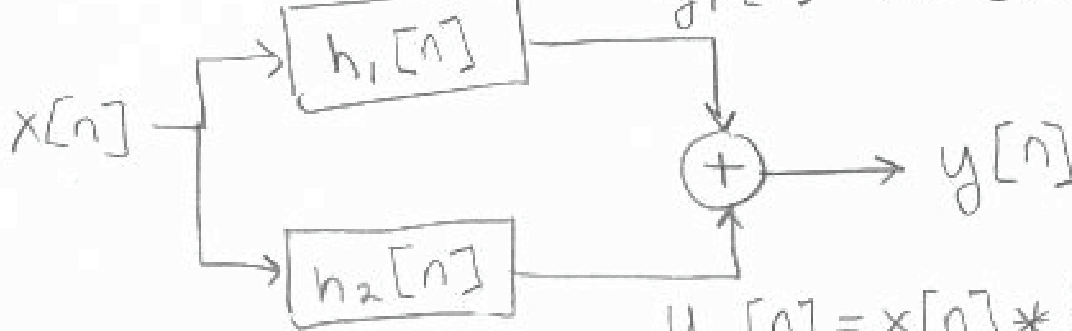
delay by 1



arbitrary impulse response $h[n]$

LTI Systems are <u>distributive</u>

$y_1[n] = x[n] * h_1[n]$



$y_2[n] = x[n] * h_2[n]$

$x[n] \rightarrow \boxed{h_1[n] + h_2[n]} \rightarrow y[n] = x[n] * \left[h_1[n] + h_2[n]\right]$

LTI systems are <u>Associative</u>

$$x[n] \rightarrow \boxed{h_1[n]} \rightarrow \boxed{h_2[n]} \rightarrow y[n]$$

$$x[n] \rightarrow \boxed{h_1[n] * h_2[n]} \rightarrow y[n]$$

$$y[n] = (x[n] * h_1[n]) * h_2[n]$$
$$= x[n] * (h_1[n] * h_2[n])$$

also, by commutative property

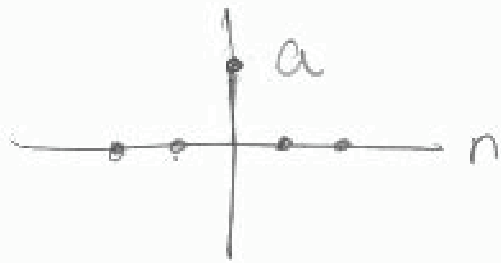
$$x[n] \rightarrow \underbrace{\boxed{h_2[n]} \rightarrow \boxed{h_1[n]}}_{\text{can switch the order!}} \rightarrow y[n]$$

Some LTI systems are <u>memory-less</u>

$$h[n] = 0 \text{ for } n \neq 0$$

$$\Downarrow$$

$$h[n] = a\delta[n]$$



$a = 1$, piece of wire

$a > 1$, amplifier

$a < 1$, attenuator

a can also be < 0

$$x[n] \rightarrow \boxed{a\delta[n]} \rightarrow y[n]$$

or simply

$$x[n] \xrightarrow{a} y[n]$$

$$y[n] = a\,x[n]$$

Some LTI Systems are causal

causality exists if

$h[n] = 0$ for $n < 0$

$$y[n] = \sum_{k=-\infty}^{n} x[k]\, h[n-k]$$

only input up to the present

only values ≥ 0

---

or for continuous

$h(t) = 0$ for $t < 0$

$$y(t) = \int_{-\infty}^{t} x(\tau)\, h(t-\tau)\, d\tau$$

---

for a signal, this is called "initial rest"