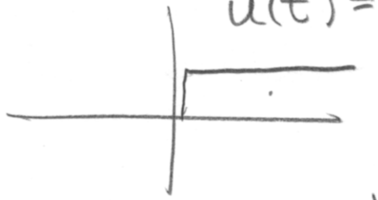


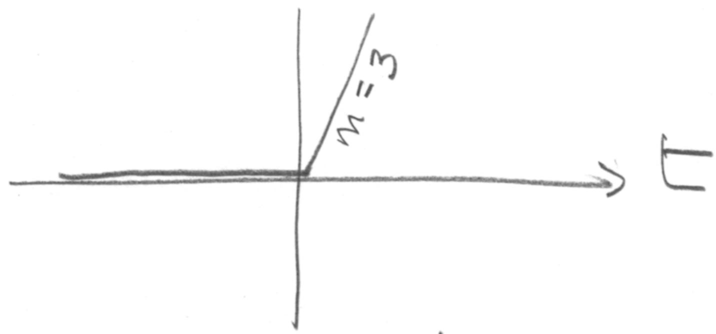
$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$

integrator has $h(t) = u(t)$

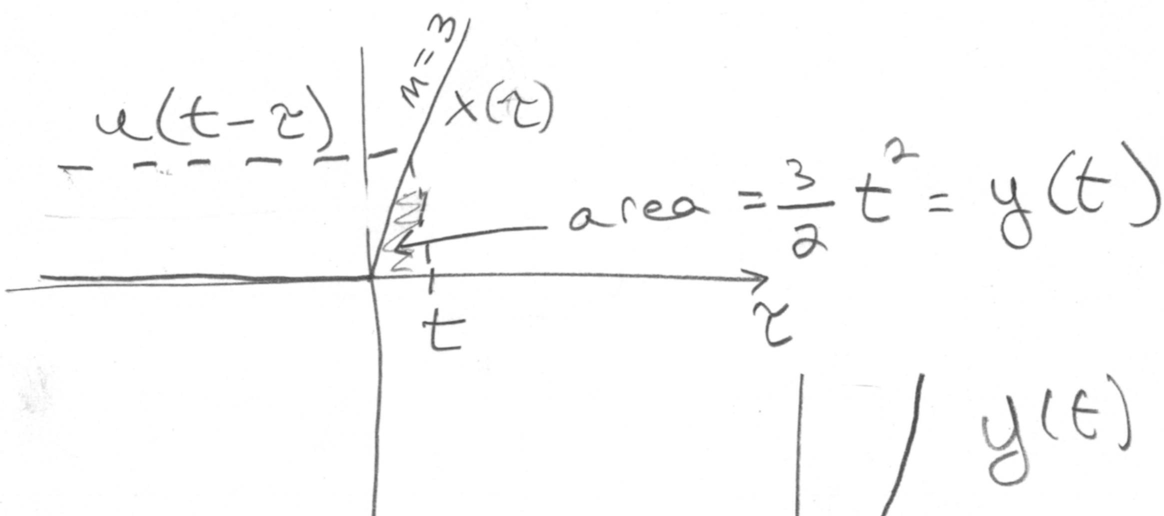
$$x(t) \rightarrow \boxed{\int} \rightarrow y(t)$$

$$x(t) \rightarrow \boxed{u(t)} \rightarrow y(t)$$

eg $x(t) = 3t\,u(t)$



$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\,u(t-\tau)\,d\tau = \int_{-\infty}^{t} x(\tau)\,d\tau = \int_{0}^{t} 3\tau\,d\tau = \frac{3}{2}t^2$$



Delay by 2

$$x[n] \longrightarrow \boxed{\delta[n-2]} \longrightarrow y[n]$$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,\underbrace{\delta[n-2-k]}_{\text{fires at } k=n-2} = x[n-2]$$

some systems are

Invertible

$$x(t) \to \boxed{h_1(t)} \xrightarrow{y(t)} \boxed{h_2(t)} \to x(t)$$

$$y(t) = x(t) * h_1(t)$$

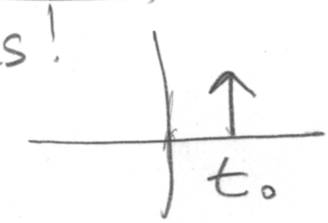
$$y(t) = 2(x(t)) \quad \text{yes!}$$

$$x(t) = \frac{1}{2} y(t)$$

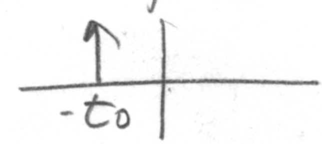
$$h_1(t) * h_2(t) = \delta(t)$$

$$y(t) = x(t - t_0) \quad \text{yes!}$$

$$h_1(t) = \delta(t - t_0)$$



$$h_2(t) = \delta(t + t_0)$$



$$\delta(t - t_0) * \delta(t + t_0) =$$

$$\delta(t)$$

$$y(t) = 3 \quad \text{no!}$$

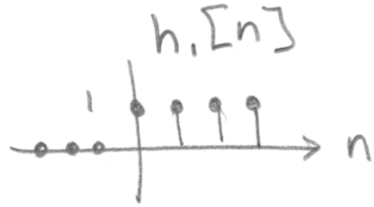
you have lost all information about $x(t)$

Another well known invertable system

$$\left(\text{such that} \quad h_1(t) * h_2(t) = \delta(t) \right)$$

is

$h_1[n] = u[n]$

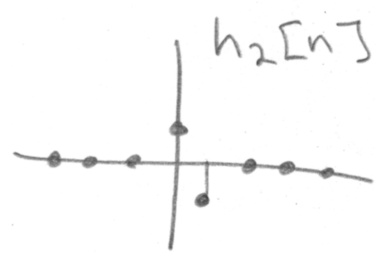


$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, u[n-k] = \sum_{k=-\infty}^{n} x[k]$$

convolving with $u[n]$ produces integration!

$h_2[n] = \delta[n] - \delta[n-1]$ today minus yesterday

convolving with $h_2[n]$ produces differentiation!

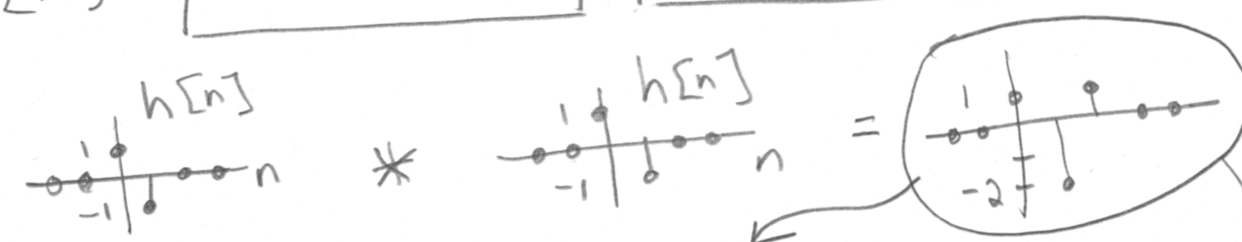


$$h_1[n] * h_2[n] = \delta[n]$$

(do it graphically)

Taking the 1rst difference twice produces the second difference. Each is a convolution, and can be combined into a single convolution

$$x[n] \rightarrow \boxed{\overset{h[n]}{\delta[n]-\delta[n-1]}} \rightarrow \boxed{\overset{h[n]}{\delta[n]-\delta[n-1]}} \rightarrow y[n]$$



$$x[n] * \left(h[n] * h[n]\right) = y[n]$$

takes the second difference

LTI systems can be described by

$\underbrace{\text{Linear Constant Coefficient}}_{\substack{\text{time invariant} \\ \text{(or space invariant} \\ \text{for images)}}}$ "Diff." Eq

can be either

"Differential" Eq. (continuous)

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

or

"Difference" Eq. (discrete)

$$\sum_{k=0}^{N} a_k \, y[n-k] = \sum_{k=0}^{M} b_k \, x[n-k]$$

Anything more than zeroth order ($a_k \neq 0$ or $b_k \neq 0$ for some $k > 0$) means the system has <u>memory</u>

All real systems have memory, e.g. inertia, electric or magnetic field, chemical concentration....

memory

$$x(t) \longrightarrow \boxed{} \longrightarrow y(t)$$

$$x[n] \longrightarrow \boxed{} \longrightarrow y[n]$$

System has memory if

$y(t)$ is effected by either $x(\tau)$ or $y(\tau)$, $\tau \neq t$

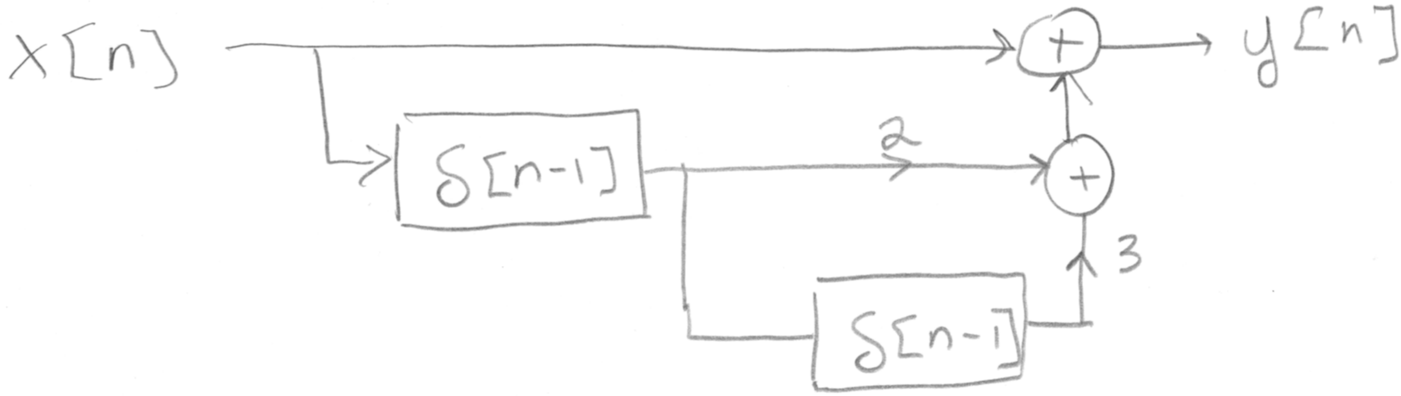
$y[n]$ is effected by either $x[k]$ or $y[k]$ $k \neq n$

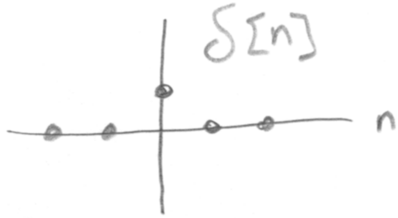
diff eq. interpretation

physical interpretation

Some LTI systems have a Finite Impulse Response (FIR)

$$y[n] = x[n] + 2x[n-1] + 3x[n-2]$$

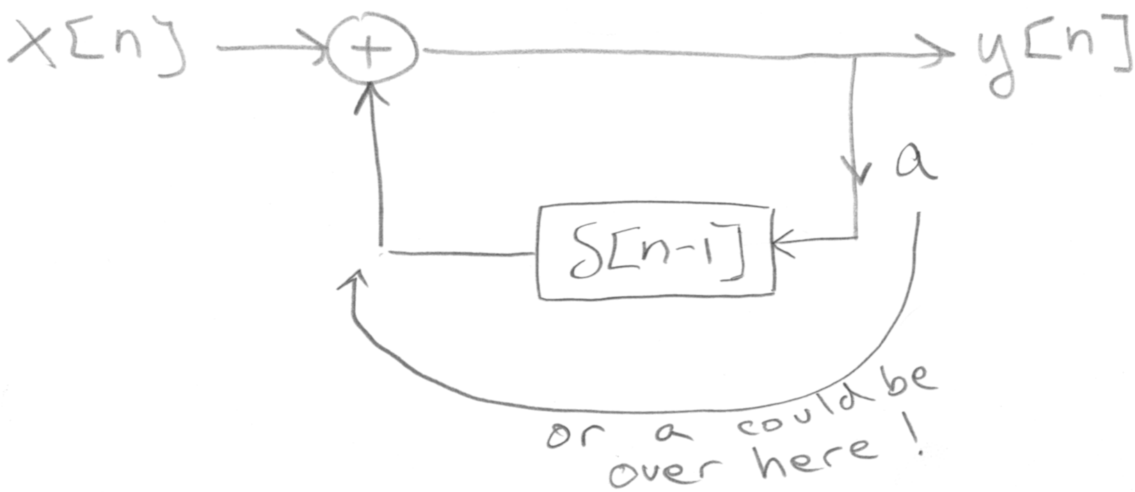




$h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]$

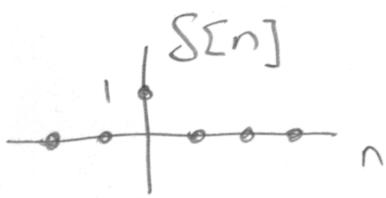
Some LTI systems have an Infinite Impulse Response (IIR)

$$y[n] = x[n] + a\,y[n-1]$$

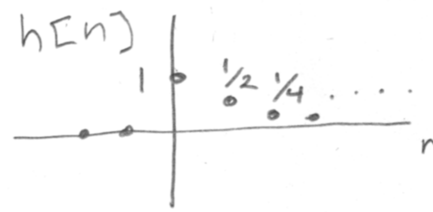


Now we need auxillary conditions, such as "initial rest"

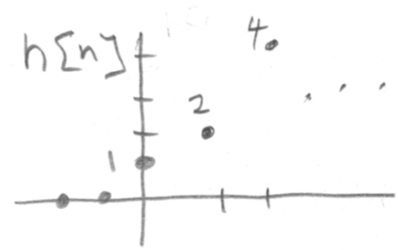
$$y[n] = 0 \quad n < 0$$



$a = \frac{1}{2}$



unstable → $a = 2$



Real Exponentials

$a = -1$

Imaginary Exponentials