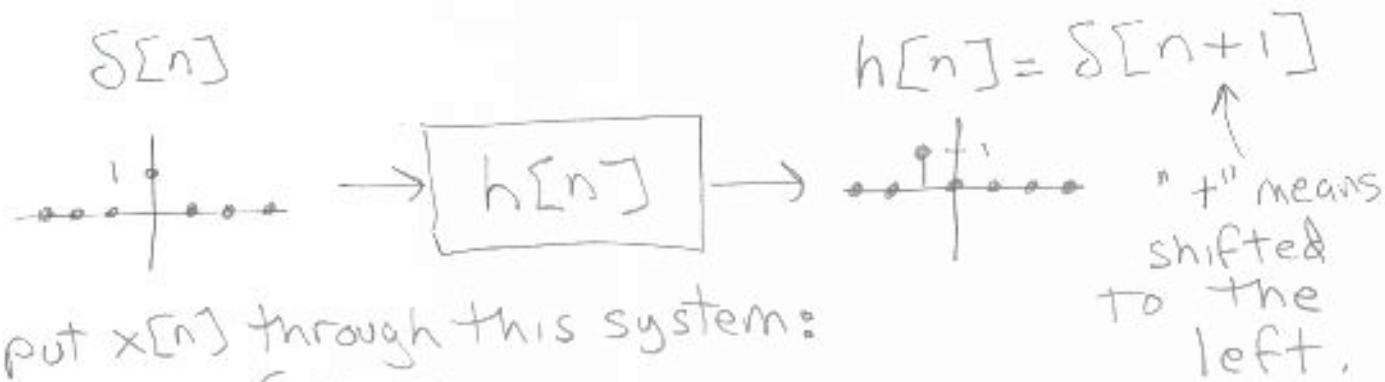


Example

A system that looks one sample into the future has what impulse response? IIR or FIR



put $x[n]$ through this system:

$$x[n] * \delta[n+1] =$$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k] \delta[n+1-k] = x[n+1]$$

↑
fires at
 $k = n+1$

sifting

\uparrow
 $x[n]$ shifted to the
left by 1,
one sample into
the future.

$$x[n] \rightarrow \boxed{\delta[n+1]} \rightarrow y[n] = x[n+1]$$

\uparrow
this system is not causal

but is FIR

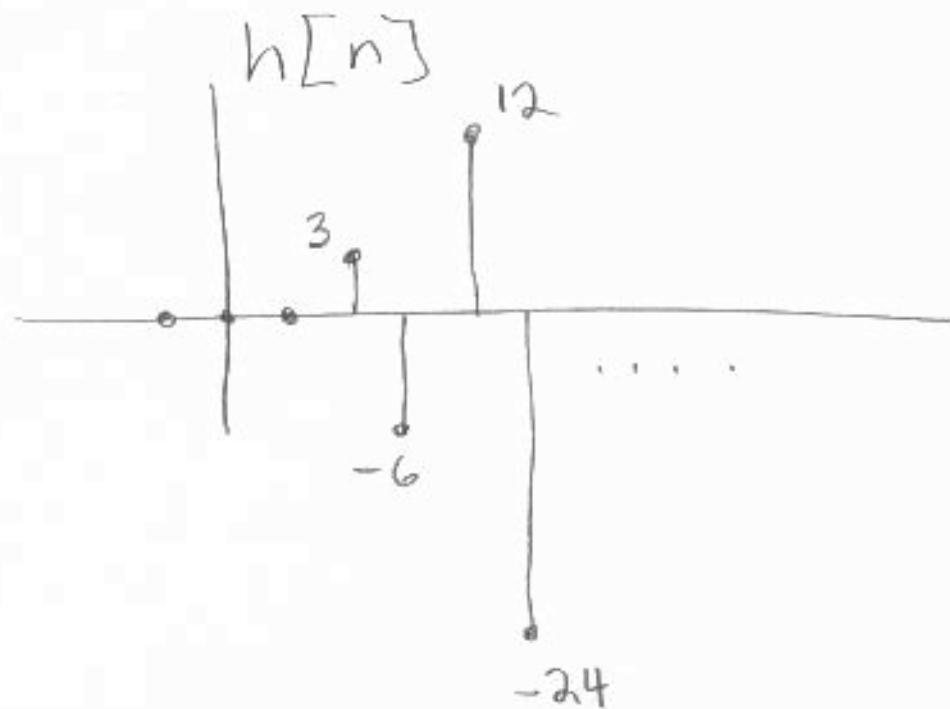
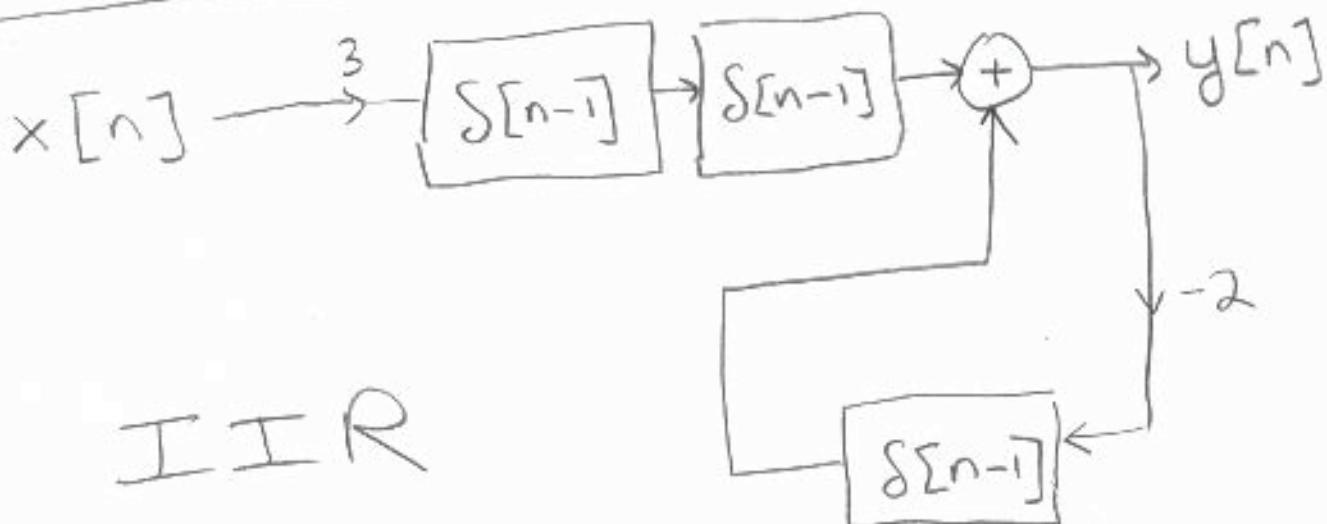
EXAMPLE

DRAW a systems diagram for

$$y[n] = 3x[n-2] - 2y[n-1]$$

IS it IIR or FIR?

what is the impulse response?



Infinite sum Formula (very useful general solution)

$$\sum_{k=0}^{\infty} a^k, \quad 0 < a < 1 = \frac{1}{1-a}$$

e.g., when $a = \frac{1}{2}$

$$k = 0 \quad 1 \quad 2 \quad 3 \dots$$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \dots = 2$$

Finite Sum Formula

$$\sum_{k=0}^n a^k, \quad 0 < a < 1 = \frac{1-a^{n+1}}{1-a}$$

e.g., when $a = \frac{1}{2}, n = 1$

$$\frac{1 - \frac{1}{4}}{1 - \frac{1}{2}} = \frac{\frac{3}{4}}{\frac{1}{2}} = 1\frac{1}{2}$$

~~makes
change sign
with every
sample if
 $a < 0$~~

Don't Bother including this

Fourier Series - periodic signals
we will only treat continuous time here
periodic means

$$x(t) = x(t + T_0) \text{ for all } t$$

also true for $2T_0, 0T_0, -17T_0$
i.e. any multiple periods

so T_0 defined as the minimum value for which this is true.

$T_0 \equiv$ fundamental period

$$\omega_0 = 2\pi/T_0 \rightarrow \text{fundamental frequency}$$

$$f_0 = 1/T_0 \rightarrow \text{fundamental frequency}$$

$$\omega_0 = 2\pi f_0$$

eg 1 $x(t) = \cos(\omega_0 t) = \cos(2\pi f_0 t)$



eg 2 $x(t) = e^{j\omega_0 t} = e^{j\frac{2\pi}{T_0} t}$

k^{th} harmonic of periodic signal

$$e^{jkw_0 t}$$

frequency
of
harmonic

K

$\circ \leftarrow \text{"D.C."}$ (direct current)
average value

$\oplus 1$ "fundamental"

or

"1st harmonic"

need both
to make

Signal real

$\oplus 2$ "2nd harmonic"

; etc

caution:

some people call
this "first harmonic"

Fourier series relates $x(t)$ to a set of complex coefficients a_k for each of the harmonics

$$x(t) \xleftrightarrow{F_s} a_k$$

Synthesis

=

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jkw_0 t}$$

analysis

=

$$a_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jkw_0 t} dt$$

notation means

over
any
period

t to $(t+T_0)$

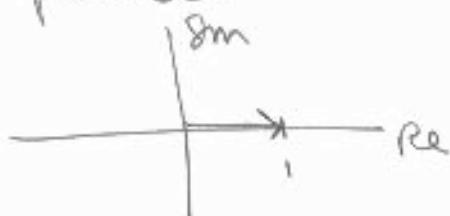
true transform, no information lost

a_0 is the average value of $x(t)$

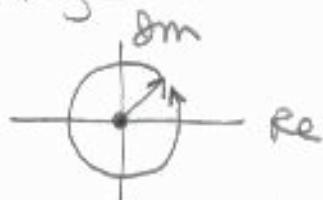
$$a_0 = \frac{1}{T_0} \int_{T_0}^1 x(t) e^{-j\omega_0 t} dt$$

$$= \frac{1}{T_0} \int_{T_0}^1 x(t) dt$$

$e^{-j\omega_0 t}$ is a stationary phasor



all other phasors $k \neq 0$, spin around the origin and thus have an average value of 0



Likewise, the sinusoids in any but the zeroth harmonic have an average value of zero.

LETS SOLVE FOR THE
FOURIER SERIES OF
SIN AND COS BY
WORKING BACKWARDS
FROM SYNTHESIS EQUATION.

THE FOURIER SERIES IS AN
ORTHOGONAL BASIS SET,
IF WE FIND ONE THAT
WORKS IT IS THE
ONLY (RIGHT) ONE.
=====

$$x(t) = \cos \omega_0 t$$

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{j k \omega_0 t}$$

$$\cos \omega_0 t = \frac{e^{j \omega_0 t} + e^{-j \omega_0 t}}{2}$$

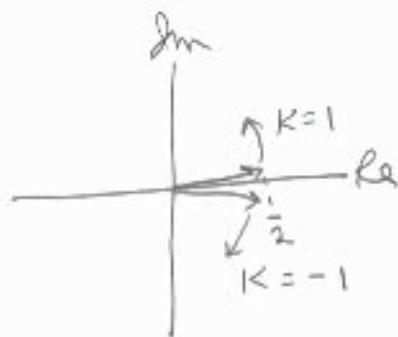
Since $e^{j \omega_0 k t}$ are "orthogonal" for all k

$$a_0 = 0$$

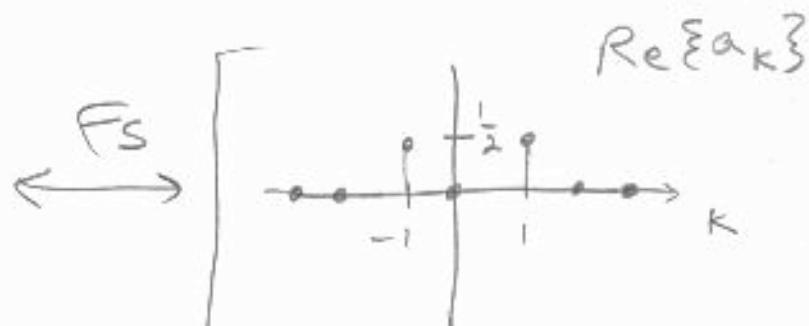
$$a_1 = \frac{1}{2}$$

$$a_{-1} = \frac{1}{2}$$

$$a_k = 0 \quad |k| > 1$$



continuous



discrete

Fourier Series
of
cos
is even and real



$$x(t) = \sin \omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$$

separate real + imaginary

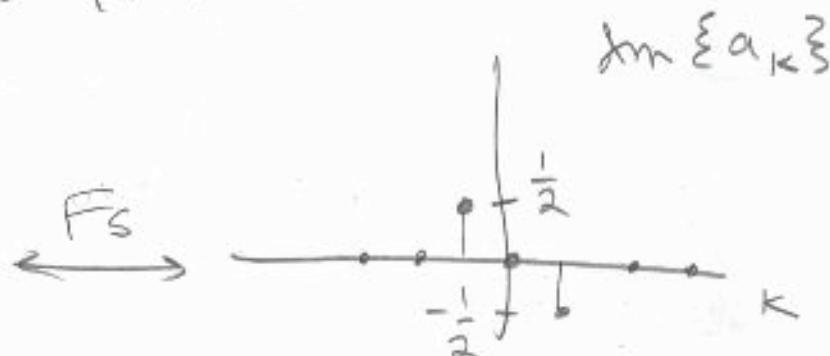
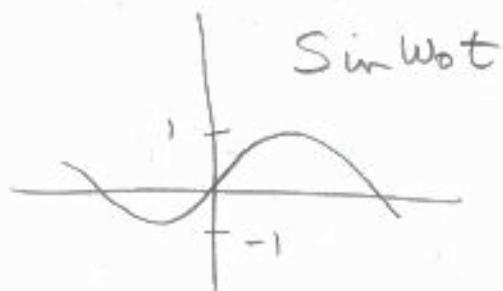
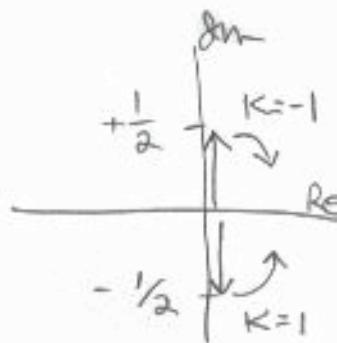
$$= -\frac{j}{2} e^{j\omega_0 t} + \frac{j}{2} e^{-j\omega_0 t}$$

$$a_0 = 0 \quad \text{no "D.C."}$$

$$a_1 = -\frac{j}{2}$$

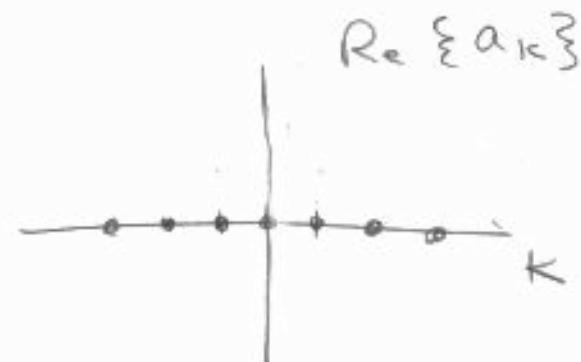
$$a_{-1} = \frac{j}{2}$$

$$a_K = 0 \quad |K| > 1$$



Fourier Series
of
Sin

is odd and imaginary



$$\text{linearity} \quad x(t) \xleftrightarrow{\text{Fs}} a_K$$
$$y(t) \xleftrightarrow{\text{Fs}} b_K$$

$$Ax(t) + By(t) \xleftrightarrow{\text{Fs}} Aa_K + Bb_K$$

time scaling

$$x(at) \xrightarrow{F_s} a_k$$

only ω_0 has changed

still have all the same magnitudes and phases for the harmonics. Just a different frequency.

a_k is a function of harmonic number, k , not of absolute frequency $k\omega_0$.

ω_0 is specified separately.