Review - Fourier Series

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \qquad a_k = \frac{1}{T_0}\int_{T_0} x(t) e^{-jk\omega_0 t}\, dt$$

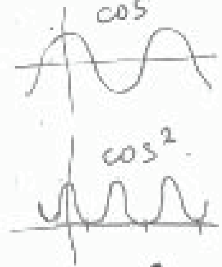
<u>synthesis</u> <u>analysis</u>

$$\frac{dx(t)}{dt} \xleftrightarrow{Fs} jk\omega_0 a_k$$

$$\int x(t)\, dt \xleftrightarrow{Fs} \frac{1}{jk\omega_0} a_k \quad \text{assumes } a_0 = 0$$

$$x(t-\tau) \xleftrightarrow{Fs} e^{-jk\omega_0 \tau} a_k \quad \text{does not change } a_0$$

multiplication

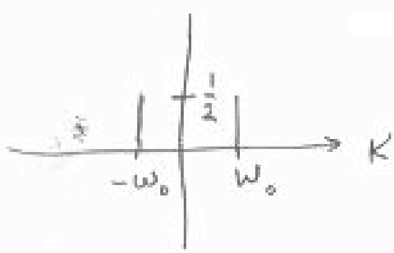


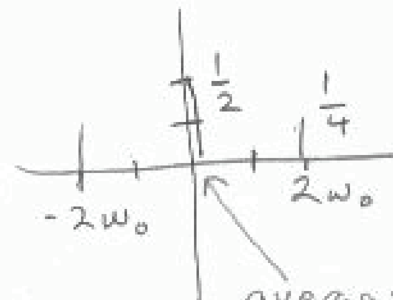
$$e^{jn\omega_0 t}\, e^{jm\omega_0 t} = e^{j(n+m)\omega_0 t}$$

e.g. $\cos(\omega_0 t)\cos(\omega_0 t) = \dfrac{1 + \cos 2\omega_0 t}{2}$

$$\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \cdot \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} = \frac{e^{2j\omega_0 t} + e^{-2j\omega_0 t}}{4} + \frac{1}{2}$$

$Re\{a_k\}$ $Re\{a_k\}$ $Re\{a_k * a_k\}$





average value is 1/2
$\sin^2 + \cos^2 = 1$

each phasor in $x(t)$ shifts each phasor in $y(t)$

convolution in frequency domain

multiplication in time domain

$$\boxed{x(t)\,y(t) \overset{F_s}{\longleftrightarrow} \sum_{n=-\infty}^{+\infty} a_n b_{k-n}}$$

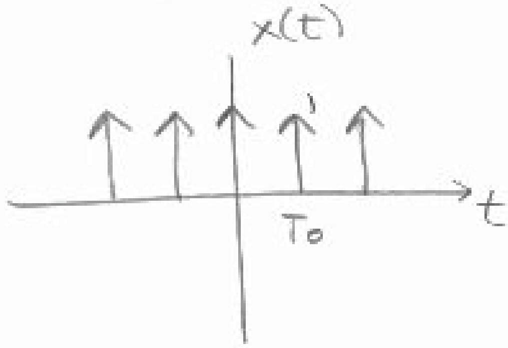
frequency shifting, superheterodyning

Armstrong

Example

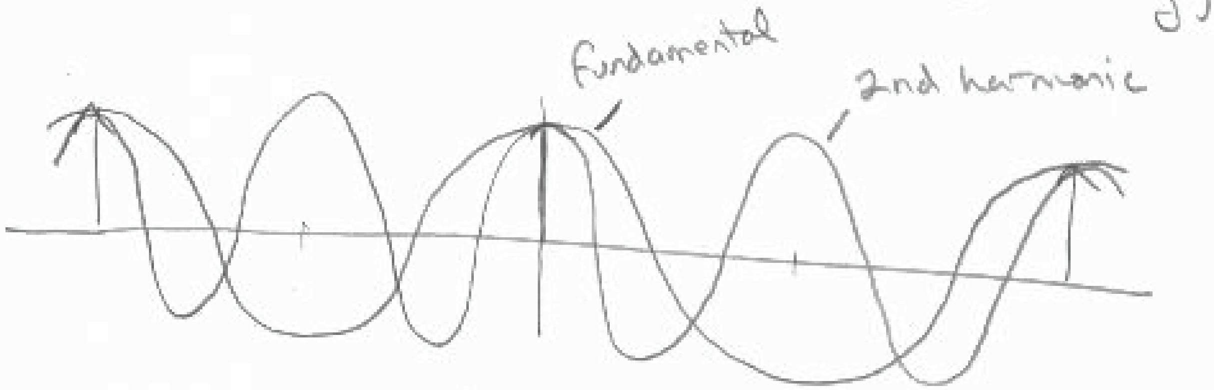
Impulse train

$$x(t) = \sum_{t=-\infty}^{+\infty} \delta(t - mT_0) \xleftrightarrow{FS} a_K = \frac{1}{T_0}, \text{ all } K$$



$$a_K = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{+\frac{T_0}{2}} \delta(t) e^{-jK\omega_0 t} \, dt = \frac{1}{T_0}$$

(sifting)



Example

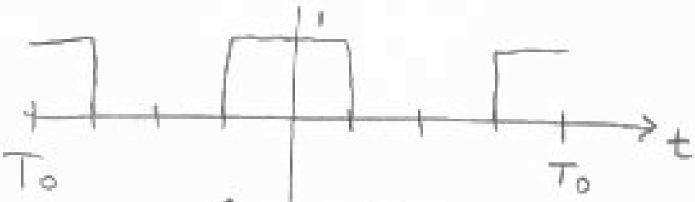
Square wave

$$x(t) = \begin{cases} 1, & |t| < \frac{T_0}{4} \\ 0, & \frac{T_0}{4} \le |t| \le \frac{T_0}{2} \end{cases}$$

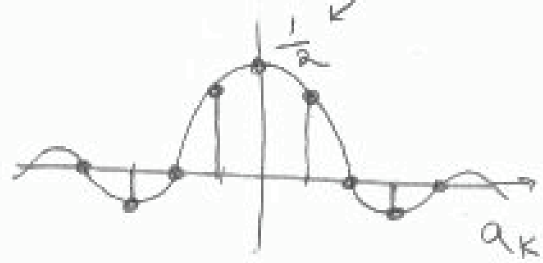
$$a_k = \frac{1}{2}\,\mathrm{sinc}\left(\frac{k}{2}\right)$$

$$x(t) = x(t + T_0)$$

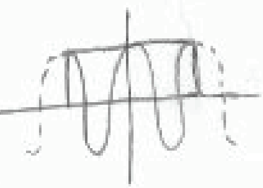
$Re\{a_k\}$



$\overset{F_S}{\longleftrightarrow}$

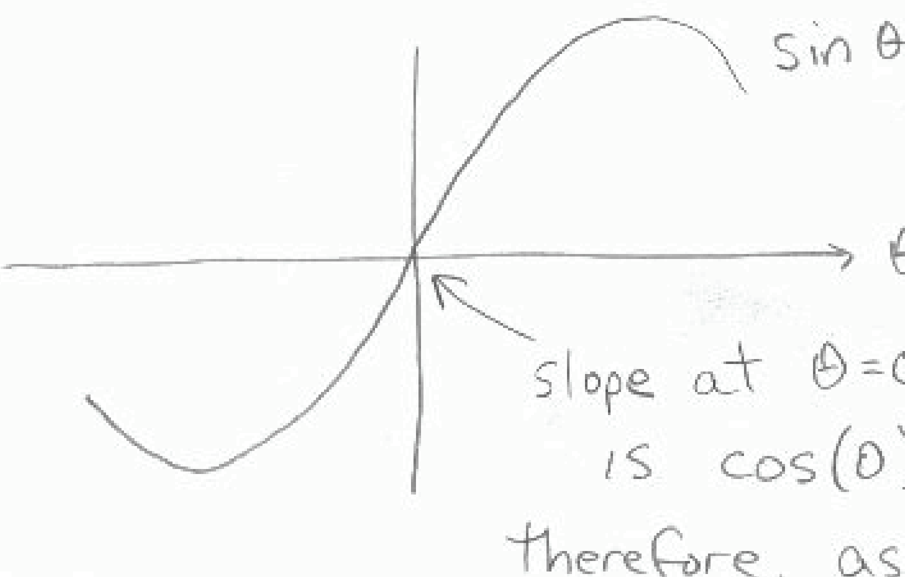


no even harmonics



$$\mathrm{sinc}\,\lambda \triangleq \frac{\sin \pi\lambda}{\pi\lambda}$$

what about sinc(0) ? $\mathrm{sinc}(0) = 1$



slope at $\theta = 0$
is $\cos(0) = 1$
therefore, as $\theta \to 0$

$$\frac{\sin\theta}{\theta} \to 1$$

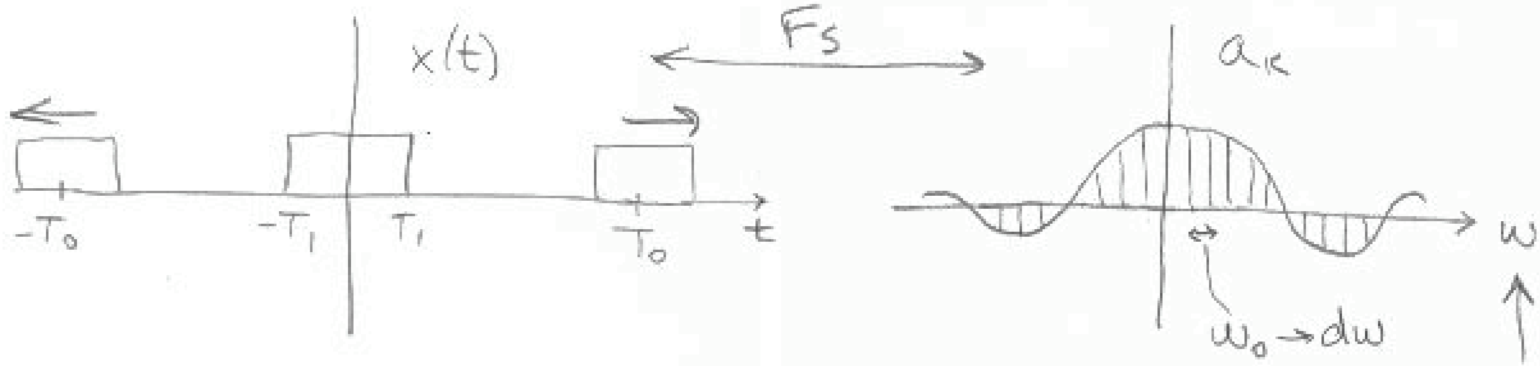
L'Hopital's Rule

L'Hopital's Rule

If $\lim_{x \to c} f(x) = 0$ and $\lim_{x \to c} g(x) = 0$

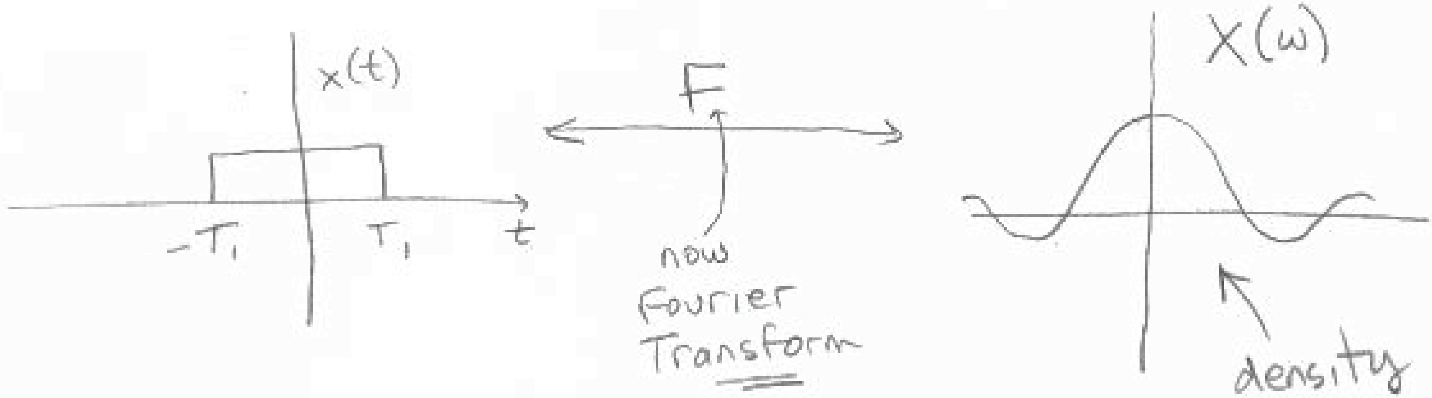
then $\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$

(same true if
$\lim_{x \to c} f(x) = \infty$ and $\lim_{x \to c} f(x) = \infty$)



no longer "K"

$$\omega_0 = \frac{2\pi}{T_0} \quad \text{as } T_0 \to \infty,\ \omega_0 \to 0$$



	Synthesis	Analysis
Series	$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$a_k = \frac{1}{T_0}\int_{T_0} x(t) e^{-jk\omega_0 t}\, dt$
Transform	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega) e^{j\omega t}\, d\omega$	$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t}\, dt$

notation

we use $X(\omega)$, so does Hsu (schaum's)

Oppenhiem uses $X(j\omega)$