

power / energy comes from ohms law



$$\frac{V}{R} = I$$

$$IR = V$$

force • distance / time

$$\text{power } P = V I = \frac{V^2}{R} = I^2 R$$

$$\text{energy } E = \int P dt$$

how much gas  
is in the  
generator

$$\text{instantaneous power } P = \frac{dE}{dt}$$

how bright  
is  
the light bulb

Bacon talks about

average power

average power

$$P_{\text{average}} = \frac{\Delta E}{\Delta t}$$

how much gas was  
used over the  
entire hour.

"ENERGY" & "POWER" - terms used

ENERGY

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt$$

$$E = \sum_{n=-\infty}^{+\infty} |x[n]|^2$$

for signals in general, even when not voltages or currents.

"| |" means magnitude, or modulus, of complex number.

For Periodic Signals

$$x(t) = x(t + T_0)$$

Period

$$x[n] = x[n + N_0]$$

Reduces to absolute value for real number.

$E = \infty$  for periodic signals where  $x(t) \neq 0$  or  $x[n] \neq 0$ , but such signals still can have finite power.

Power

$$P = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt$$

integrate over any single period.

$$P = \frac{1}{N_0} \sum_{n=1}^{N_0} |x[n]|^2$$

sum over any single period.

# Parseval's relation - average power P

$$P = \frac{1}{T_0} \int_{-T_0}^{+T_0} |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

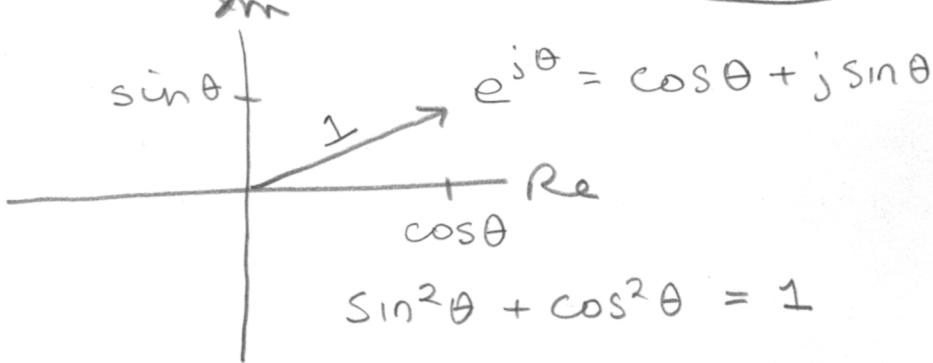
$|a_k|$  does not depend on phase

$$a_k = A_k e^{j\theta_k}$$

complex ↑      real ↑

$$|a_k| = A_k \text{ because } |e^{j\theta_k}| = 1$$

$$|e^{j\theta_k}| = 1$$



average power P does not depend on  $\omega_0$ !

shorter period  $T_0 \Rightarrow$  less energy / cycle

but same average power

not like light,  $h\nu$ , where short wavelengths have higher energy

Contribution of each harmonic to P

is independent! They do not effect each other. Why?

I don't know, you tell me.

example

what is average power of

$$x(t) = \cos(2\pi t) + \sin(4\pi t)$$

↑  
Fundamental  
(1st Harmonic)      ↑  
Second Harmonic

$$\omega_0 = 2\pi$$

$$a_0 = 0$$

$$a_1 = \frac{1}{2}$$

$$a_{-1} = \frac{1}{2}$$

$$a_2 = -\frac{j}{2}$$

$$a_{-2} = \frac{j}{2}$$

$$a_k = 0, \text{ all other } k$$

$$P = \sum_{k=-\infty}^{+\infty} |a_k|^2 = \left|\frac{1}{2}\right|^2 + \left|\frac{1}{2}\right|^2 + \left|-\frac{j}{2}\right|^2 + \left|\frac{j}{2}\right|^2 = 1$$

# Parseval's relation for Fourier Transform

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(\omega)|^2 d\omega$$

Energy instead of Power

↑  
Fourier  
Transform

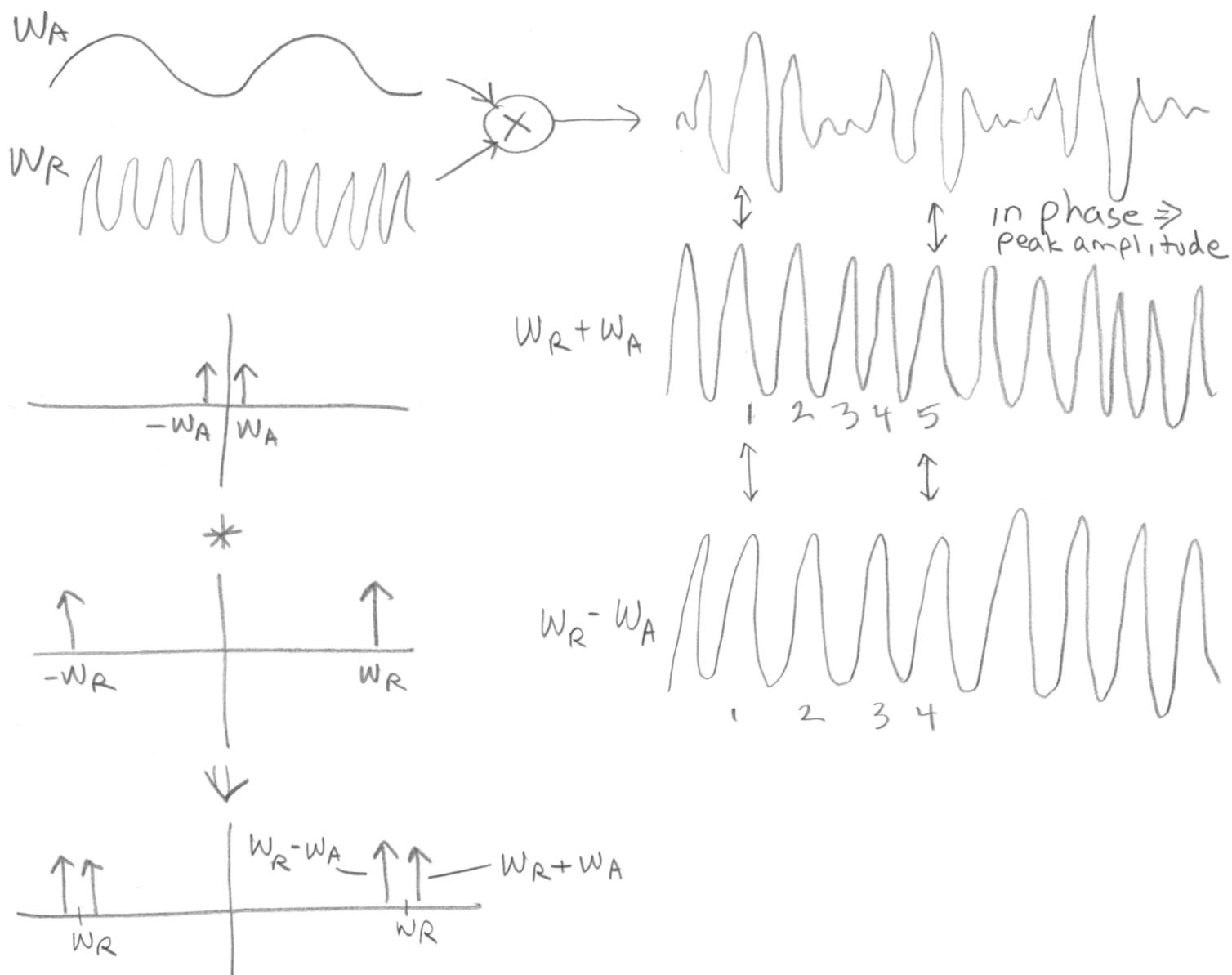
↑  
Fourier  
Series

multiplication in The time domain  
equals convolution in the frequency domain

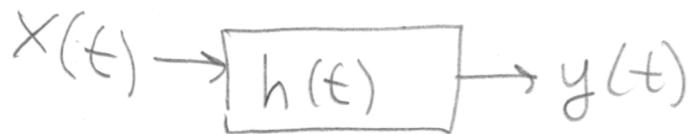
Example Amplitude Modulation (AM)

$$\cos(\omega_R t) \cos(\omega_A t)$$

↑  
Radio frequency  
 $\sim 1 \text{ MHz}$       ↑  
Audio frequency  
 $\sim 1 \text{ kHz}$



Convolution in time domain equals multiplication in frequency domain



$$y(t) = x(t) * h(t)$$

$$Y(\omega) = X(\omega)H(\omega)$$

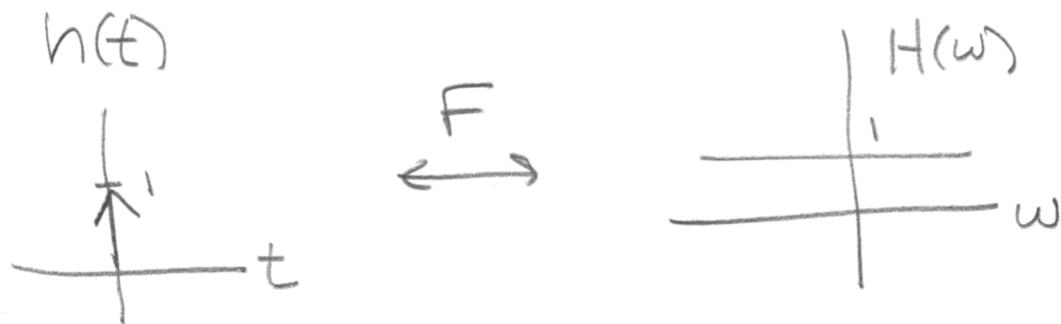
The phasor in  $x(t)$  at frequency  $\omega$  is scaled and rotated by the phasor in  $x(t)$  at frequency  $\omega$ , to become a phasor in  $y(t)$  at frequency  $\omega$ . All that changes is magnitude and phase.

example

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$h(t) = \delta(t)$$

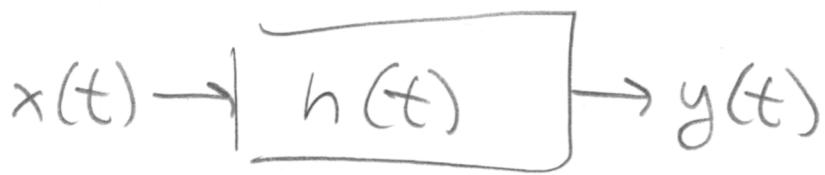
$$y(t) = x(t) * \delta(t) = x(t)$$



$$Y(\omega) = X(\omega) H(\omega) = X(\omega) 1 = X(\omega)$$

convolving with  $\delta(t)$  in the time domain is like multiplying by  $H(\omega) = 1$  in the frequency domain.  
The output simply equals the input,

$$x(t) * y(t) \xleftrightarrow{F} X(\omega)Y(\omega)$$



$$h(t) = \frac{1}{t^2}$$

perfect amplifier with gain of 2

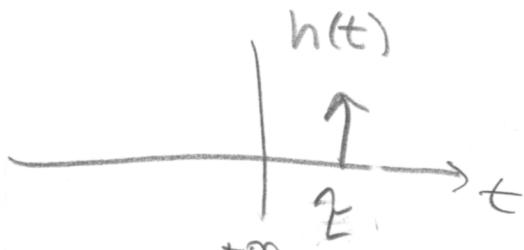
$$H(\omega) = \frac{1}{\omega^2}$$

$$Y(\omega) = H(\omega)X(\omega) = 2X(\omega)$$

all frequencies boosted by 2  
no phase shift

delay line

$$h(t) = \delta(t - \tau)$$



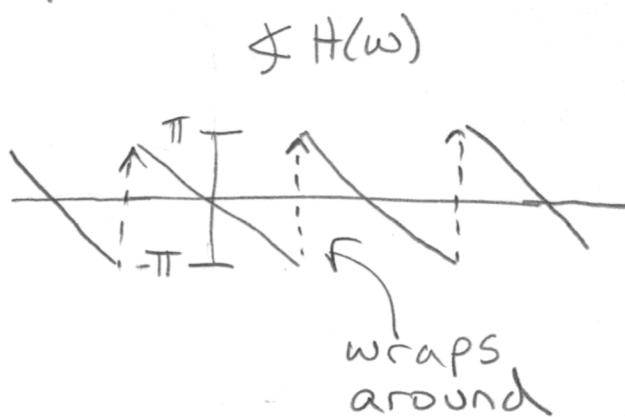
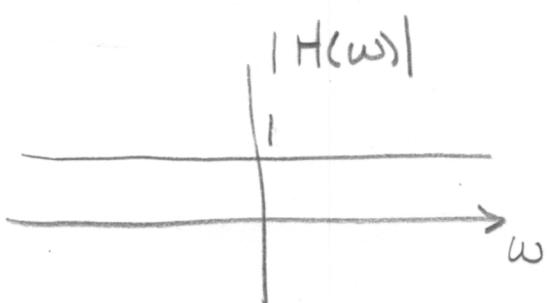
$$H(\omega) = \int_{-\infty}^{+\infty} h(t) e^{-j\omega t} dt = \int_{-\infty}^{+\infty} \delta(t - \tau) e^{-j\omega t} dt$$

sifting,  $\delta$  fires at  $t = \tau$

$$H(\omega) = e^{-j\omega\tau} \quad Y(\omega) = X(\omega) H(\omega)$$

unit gain, phase shift proportional to  $\omega$

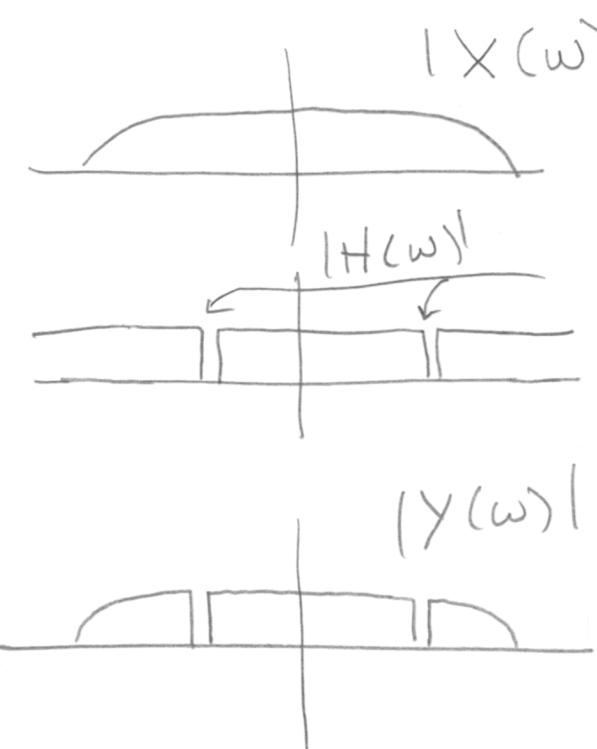
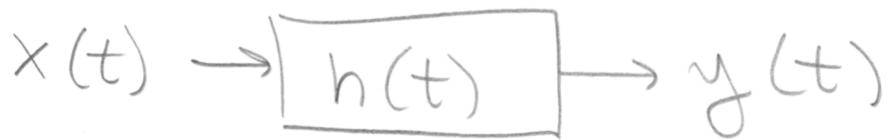
phase magnitude plot of  $H(\omega)$



$H(\omega)$  is a complex number at a given  $\omega$   
we are just plotting  $r$  and  $\theta$

$$H(\omega) = r(\omega) e^{j\theta(\omega)}$$

example



Voice (assume all cosines so  $X(w)$  is real)

microphone with missing frequencies

output also missing frequencies

So for convolution

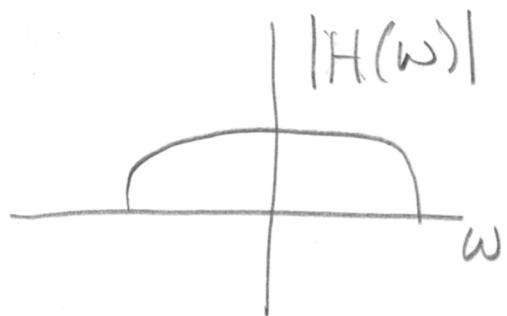
$$x(t) * y(t) \xleftrightarrow{F} X(w) Y(w)$$

# 4 general classes of filter

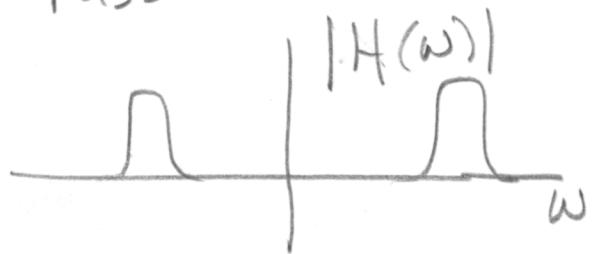
High Pass



Low Pass

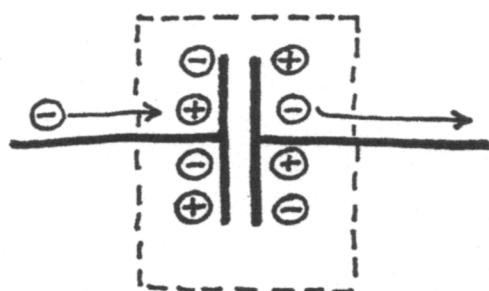
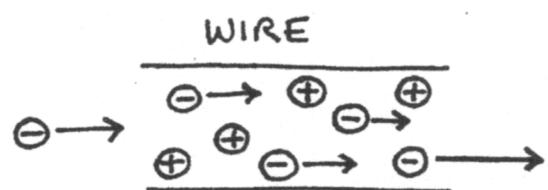
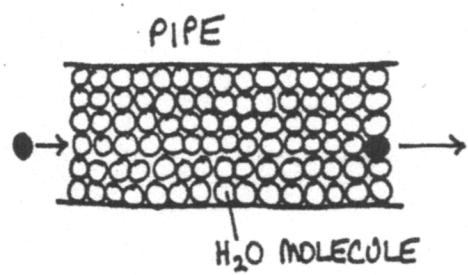


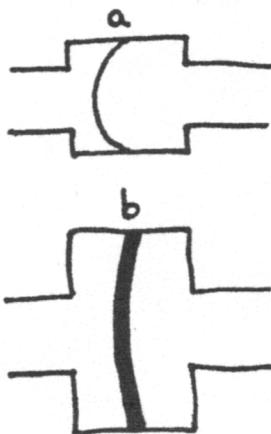
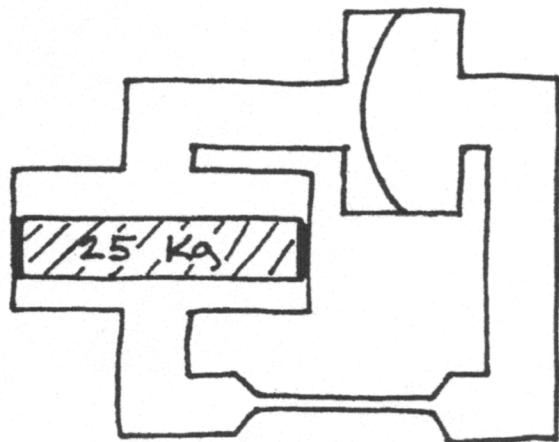
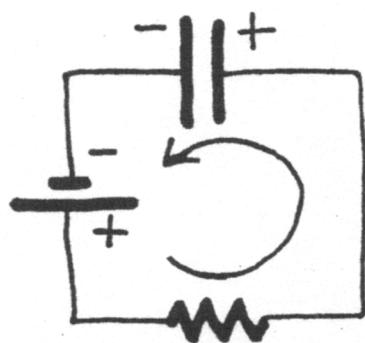
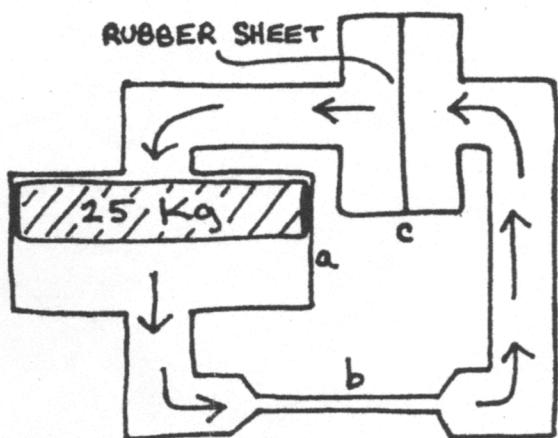
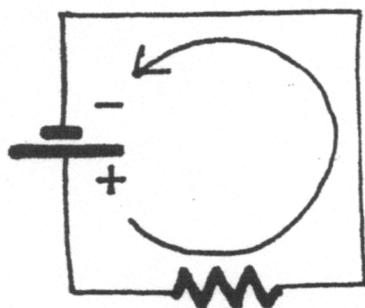
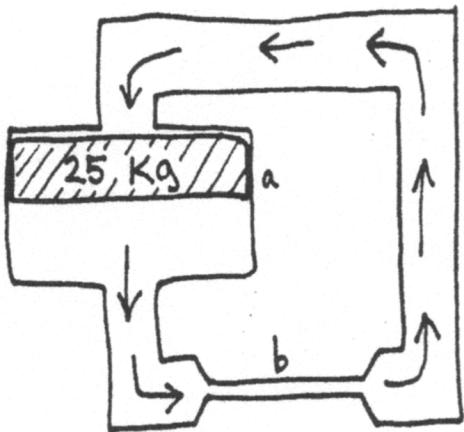
Band Pass

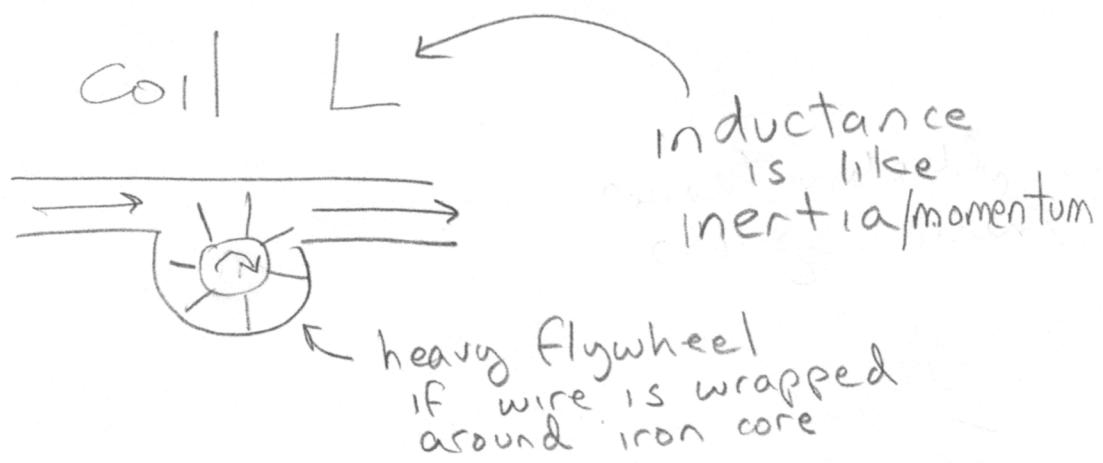


Notch (Band Block)









$$\frac{1}{T} \quad I = C \frac{dV}{dt} \quad V = \frac{1}{C} \int I dt$$

$$V = L \frac{dI}{dt} \quad I = \frac{1}{L} \int V dt$$

fig 7

PIPE

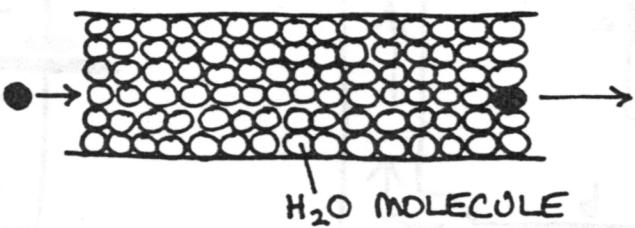


fig 8

WIRE

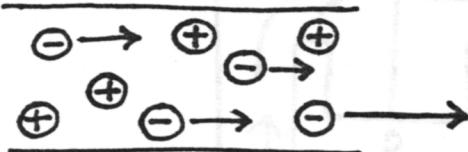


fig 9

