$$E = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt \qquad E = \sum_{n=-\infty}^{+\infty} |x[n]|^2 \quad \text{(Energy)}$$

$$x(t) = x(t - T_0) \qquad \omega_0 = \frac{2\pi}{T_0} \qquad x[n] = x[n - N_0] \quad \text{(Periodic)}$$

$$P = \frac{1}{T_0} \int_{T_0} |x(t)|^2 \, dt \qquad P = \frac{1}{N_0} \sum_{n=\langle N_0 \rangle} |x[n]|^2 \quad \text{(Power)}$$

Fourier Series

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \qquad a_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$$

$$x(t) \overset{Fs}{\longleftrightarrow} a_k \qquad y(t) \overset{Fs}{\longleftrightarrow} b_k \qquad a_k = a_{-k}^{*} \quad \text{for real } x(t)$$

$$Ax(t) + By(t) \overset{Fs}{\longleftrightarrow} Aa_k + Bb_k \qquad x(-t) \overset{Fs}{\longleftrightarrow} a_{-k}$$

$$x(\alpha t) \overset{Fs}{\longleftrightarrow} a_k, \quad \alpha > 0$$

$$\frac{dx(t)}{dt} \overset{Fs}{\longleftrightarrow} jk\omega_0 a_k \qquad \int x(t) dt \overset{Fs}{\longleftrightarrow} \frac{1}{jk\omega_0} a_k$$

$$x(t - \tau) \overset{Fs}{\longleftrightarrow} e^{-jk\omega_0 \tau} a_k$$

$$x(t) y(t) \overset{Fs}{\longleftrightarrow} \sum_{n=-\infty}^{+\infty} a_n b_{k-n}$$

$$\frac{1}{T_0} \int_{T_0} |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2 \quad \text{(Parseval's Relation)}$$

$$x(t) = 1 \overset{Fs}{\longleftrightarrow} a_0 = 1 \qquad x(t) = \sum_{n=-\infty}^{+\infty} \partial(t - nT_0) \overset{Fs}{\longleftrightarrow} a_k = \frac{1}{T_0}$$

$$x(t) = \cos(k\omega_0 t) \overset{Fs}{\longleftrightarrow} a_k = \frac{1}{2}, \; a_{-k} = \frac{1}{2} \qquad x(t) = \sin(k\omega_0 t) \overset{Fs}{\longleftrightarrow} a_k = -\frac{j}{2}, \; a_{-k} = \frac{j}{2}$$

Fourier Transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega) e^{j\omega t} d\omega$$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$x(t) \stackrel{F}{\longleftrightarrow} X(\omega)$$

$$X(\omega) = X^*(-\omega) \quad \text{for real } x(t)$$

$$Ax(t) + By(t) \stackrel{F}{\longleftrightarrow} AX(\omega) + BY(\omega)$$

$$x(-t) \stackrel{F}{\longleftrightarrow} X(-\omega)$$

$$x(\alpha t) \stackrel{F}{\longleftrightarrow} \frac{1}{\alpha} X\left(\frac{\omega}{\alpha}\right) \qquad \alpha > 0$$

$$\frac{dx(t)}{dt} \stackrel{F}{\longleftrightarrow} j\omega X(\omega)$$

$$\int x(t) dt \stackrel{F}{\longleftrightarrow} \frac{1}{j\omega} X(\omega)$$

$$x(t-\tau) \stackrel{F}{\longleftrightarrow} e^{-j\omega\tau} X(\omega)$$

$$x(t)y(t) \stackrel{F}{\longleftrightarrow} \frac{1}{2\pi} X(\omega) * Y(\omega)$$

$$x(t) * y(t) \stackrel{F}{\longleftrightarrow} X(\omega)Y(\omega)$$

$$\text{E} = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(\omega)|^2 d\omega \quad \text{(Parseval's Relation)}$$

$$X(\omega) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

(relation of Fourier Transform to Fourier Series)

$$x(t) = 1 \xleftrightarrow{F} X(\omega) = 2\pi\partial(\omega) \qquad x(t) = \partial(t) \xleftrightarrow{F} X(\omega) = 1$$

$$x(t) = \cos(\omega_1 t) \xleftrightarrow{F} X(\omega) = \pi\partial(\omega - \omega_1) + \pi\partial(\omega + \omega_1)$$

$$x(t) = \sin(\omega_1 t) \xleftrightarrow{F} X(\omega) = -j\pi\partial(\omega - \omega_1) + j\pi\partial(\omega + \omega_1)$$

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0 & |t| > T_1 \end{cases} \xleftrightarrow{F} X(\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

$$\text{sinc}\lambda = \frac{\sin\pi\lambda}{\pi\lambda}$$

Complex Impedance

$$Z_c = \frac{1}{j\omega C} \qquad Z_L = j\omega L \qquad Z_R = R$$

$$Z_S = Z_1 + Z_2 \text{ (series)} \qquad Z_P = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} \text{ (parallel)}$$

Special Cases –

Integration

what if there is a DC component $X(0) \neq 0$?
($\omega = 0$)

$$\int_{-\infty}^{t} x(\tau)\,d\tau \longleftrightarrow \pi X(0)\delta(\omega) + \frac{1}{j\omega}X(\omega)$$

not 2π, but π because integration from $-\infty$ to finite t covers $\frac{1}{2}$ the real number line

Scale

$$x(\alpha t) \overset{F}{\longleftrightarrow} \frac{1}{\alpha} X\left(\frac{\omega}{\alpha}\right)$$

this normalizing term applies even for signals with impulses in $X(\omega)$.
As shown in HSU (problem 1.25)

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

So, for example, $\cos(t) \overset{F}{\longleftrightarrow} \frac{1}{2}\delta(\omega-1) + \frac{1}{2}\delta(\omega+1)$

Scaling by $\alpha = 2$, $\cos(2t) \overset{F}{\longleftrightarrow} \frac{1}{4}\delta(\frac{\omega}{2}-1) + \frac{1}{4}\delta(\frac{\omega}{2}+1)$

These impulse may seem to be half the size they should, but setting $a = \frac{1}{2}$ we convert them, i.e., $\delta(\frac{\omega}{2}) = 2\delta(\omega)$ yielding the expected

$$\cos(2t) \overset{F}{\longleftrightarrow} \frac{1}{2}\delta(\omega-2) + \frac{1}{2}\delta(\omega+2)$$

Parsevals Relation for Fourier Transform

energy

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(\omega)|^2 d\omega$$

This will be infinite for a non-zero periodic function, which will have an impulse function in the spectrum.

$$\int_{-\infty}^{+\infty} |\delta(\omega)|^2 d\omega = \infty$$

Sifting results in on $\delta(\omega)$ taking a "snapshot" of the other.

Such functions will, however, have <u>finite</u> power. Recall that for the Fourier <u>series</u> Parsevals Relation was

$$P = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

fig 10

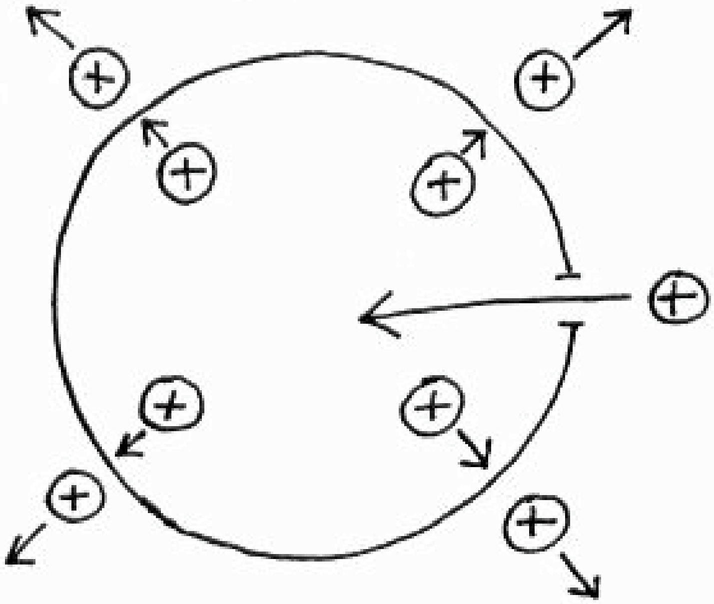


fig 11

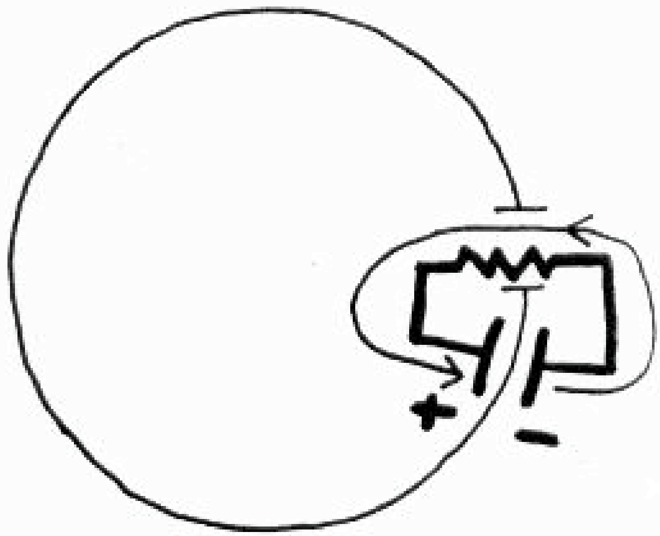


fig 12

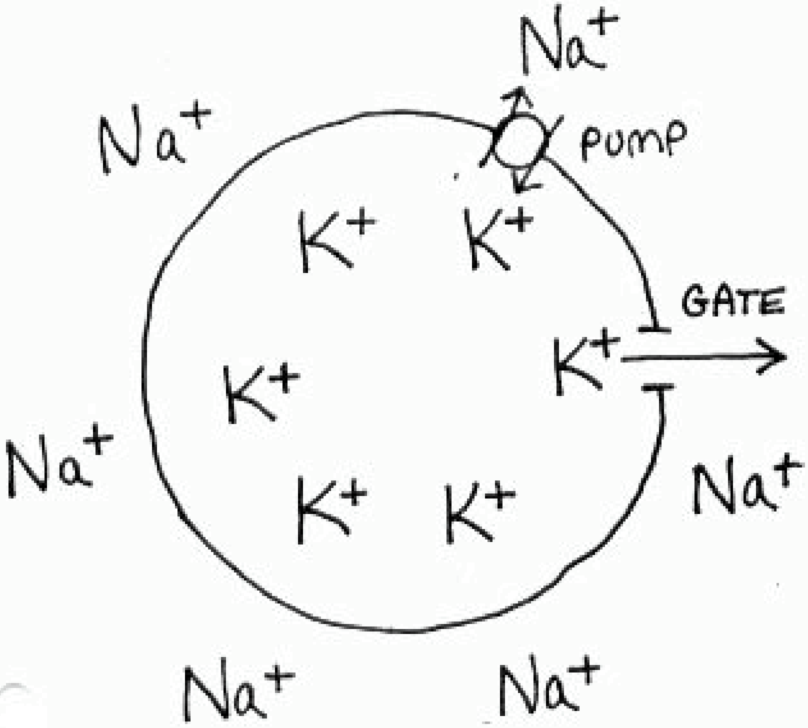
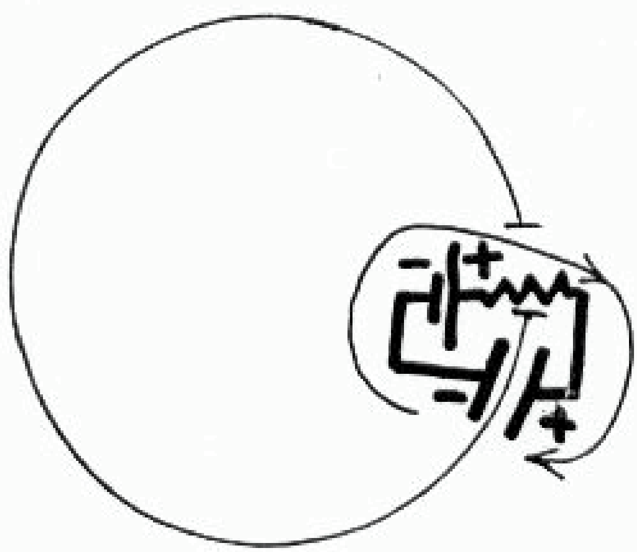
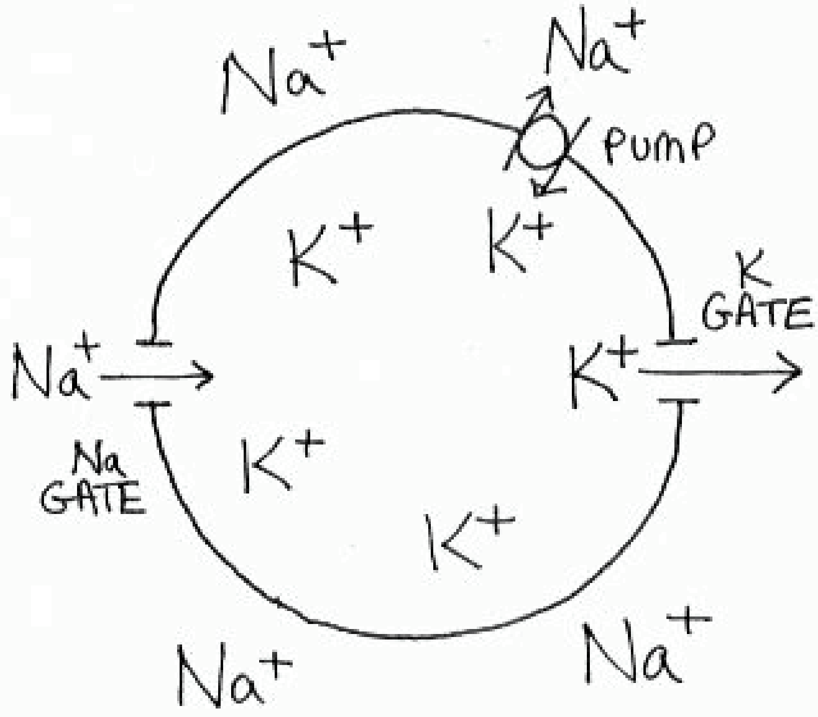
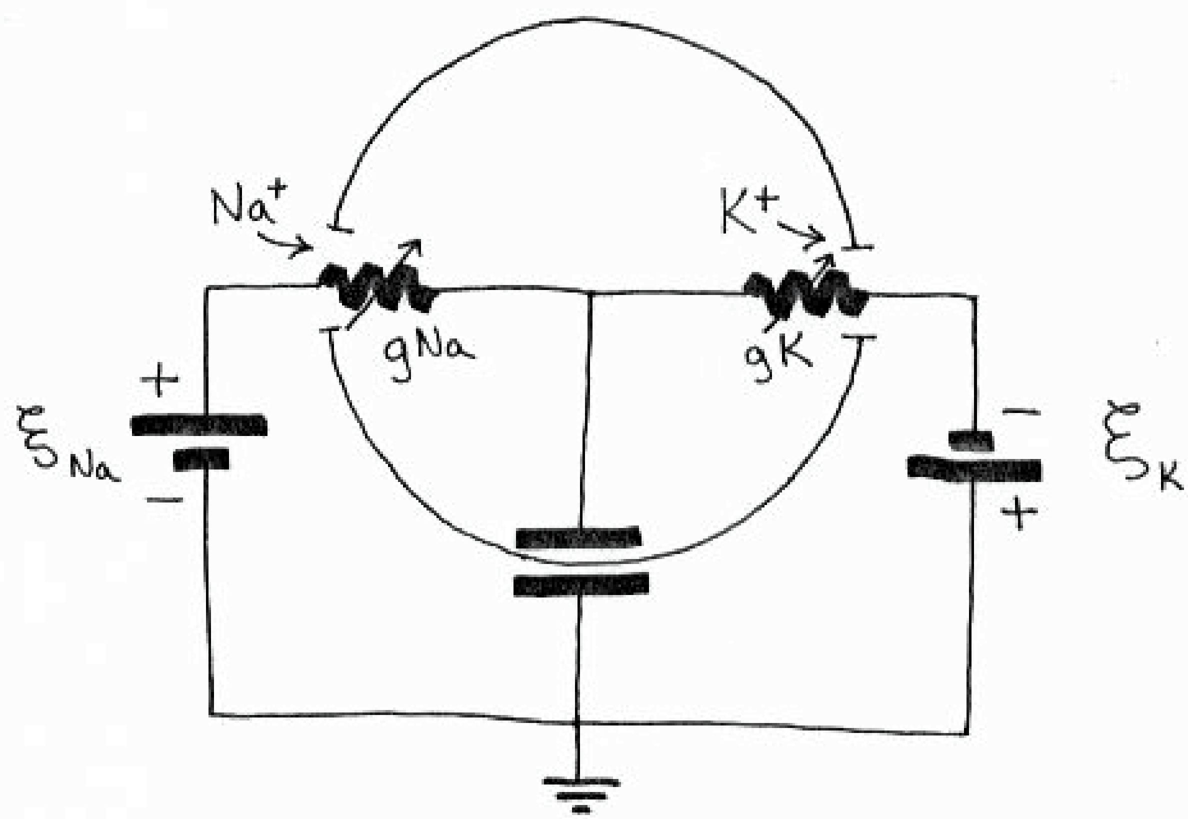


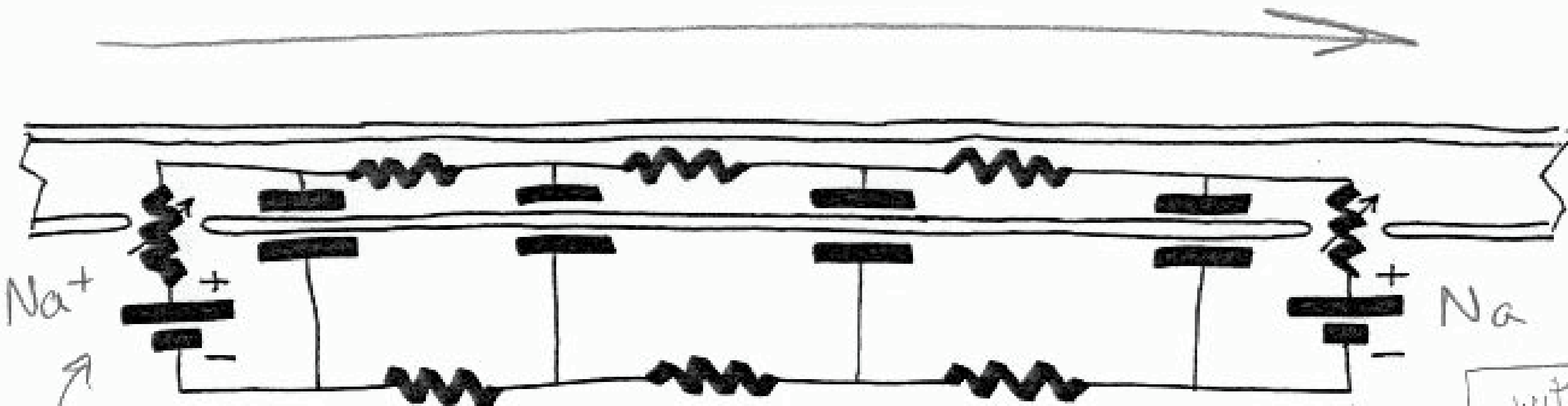
fig 13




Na+
Na+
PUMP
K+
K+
K
GATE
Na+
K+
Na
GATE
K+
K+
Na+
Na+


Na+
K+
gNa
gK
+
ξNa
−
−
ξK
+

Self propogating wave of depolarization



① this gate opens

② positive charge fills cell

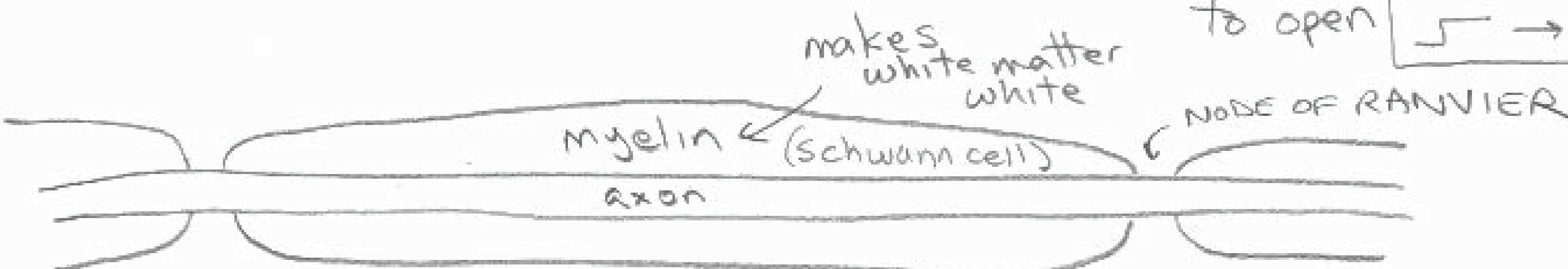
③ causes next Na+ to open

without myelin, low pass filter

→

with myelin, piece of wire

→



↓ capacitance ↑ speed, steel pipe instead of rubber hose