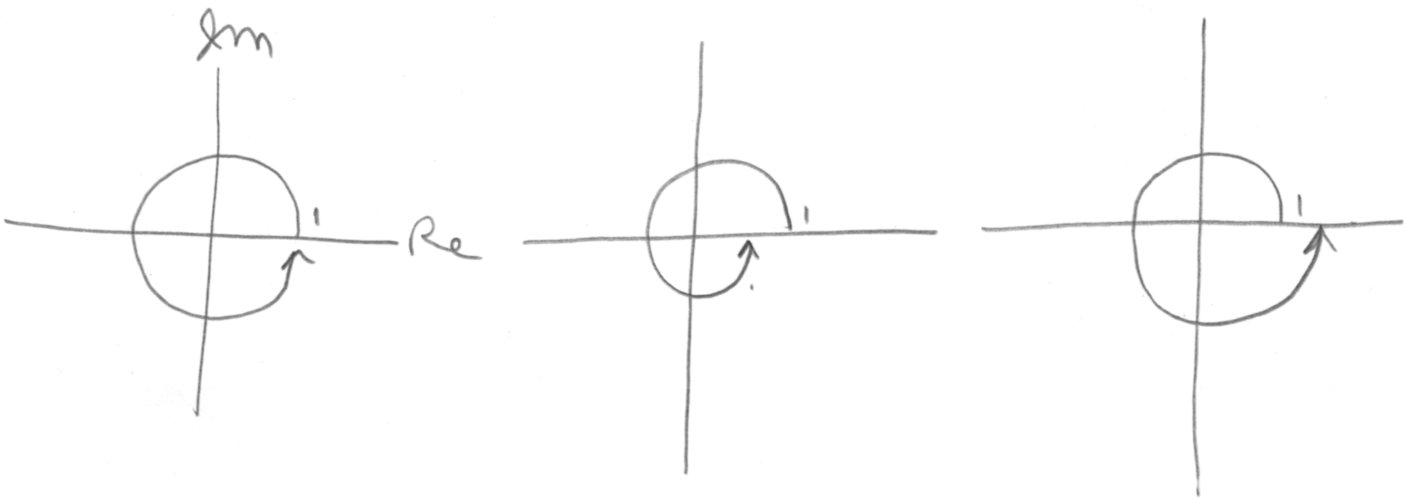
complex exponent

$$e^{(\sigma + j\omega)t} = e^{\sigma t} e^{j\omega t}$$



$\sigma = 0$	$\sigma < 0$	$\sigma > 0$
frictionless system, e.g. electron, (or $\sigma > 0$ with non-linear limit)	normal entropy, friction, resistance	needs "gain"; energy input

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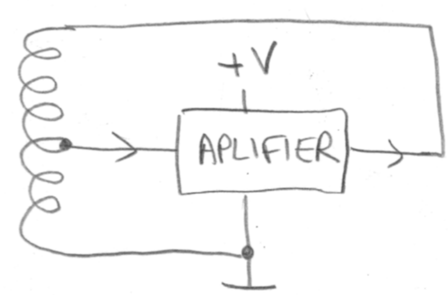
macroscopic (imperfect) oscillator:
$\sigma > 0$ with
non linear limit or friction
eg

MAGNET IN TOP

example of magic top

microscopic perfect oscillator
$\sigma = 0$ frictionless, eg:
① current in MR magnet (super conducting)



② electron in orbit xmitt and receives its own radiation

Laplace transform

$$x(t) \xleftrightarrow{\mathcal{L}} X(s) \qquad s = \sigma + j\omega$$

analysis "2 sided"

pick an "s"

$$X(s) = \int_{-\infty}^{+\infty} x(t) e^{-st}\, dt$$

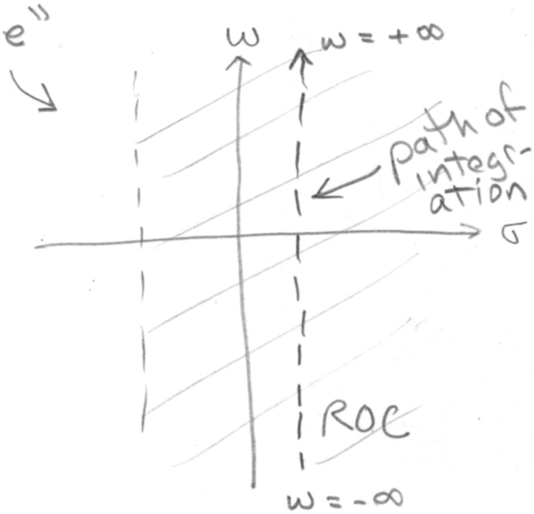
"1 sided"

$$X(s) = \int_{0-}^{+\infty} x(t) e^{-st}\, dt$$

assume $x(t) = 0,\ t<0$
or just always
multiply by $u(t)$

preserves the integral of $\delta(t)$

"s-plane"



Region of convergence

synthesis $x(t) = \dfrac{1}{2\pi j} \displaystyle\int_{\sigma - j\infty}^{\sigma + j\infty} X(s) e^{st}\, ds$

summation of exponentially
modulated phasors

different from Fourier Synthesis

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) e^{j\omega t}\, d\omega$$

since $s = \sigma + j\omega$, $ds = j\,d\omega$

when $\sigma = 0$, $s = j\omega$

$$X(s) = X(j\omega)$$

Laplace =
Fourier

we leave
the "j"
out when
denoting
Fourier,
Oppenheim
leaves it in

$$X(\sigma + j\omega) = \int_{-\infty}^{\infty} [x(t) e^{-\sigma t}] e^{-j\omega t}\, dt =$$

$$F\{x(t) e^{-\sigma t}\}$$

$x(t)$ can be "stabilized"

Let's look again at $x(t) = e^{-at}u(t)$ $a>0$

<u>Laplace Analysis</u>

only converges if $\sigma > -a$

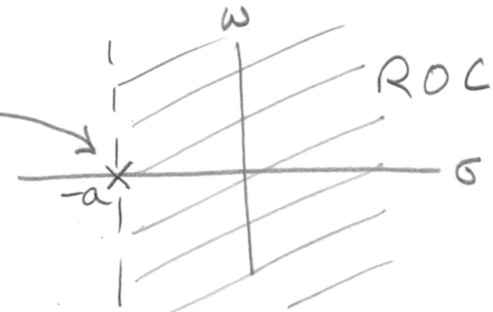
$$X(s) = \int_{-\infty}^{+\infty} x(t)e^{-st}\,dt = \int_{-\infty}^{+\infty} e^{-at}u(t)e^{-st}\,dt = \int_{0}^{\infty} e^{-(a+s)t}\,dt$$

$s = \sigma + j\omega$

0 ← "one sided"

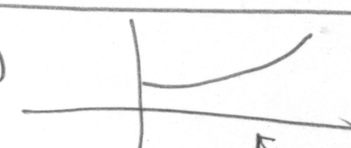
$$X(s) = -\frac{1}{a+s} e^{-(a+s)t}\Big|_0^\infty = \frac{1}{a+s} = \frac{1}{(a+\sigma)+j\omega}$$

"pole"
$X(s) = \infty$



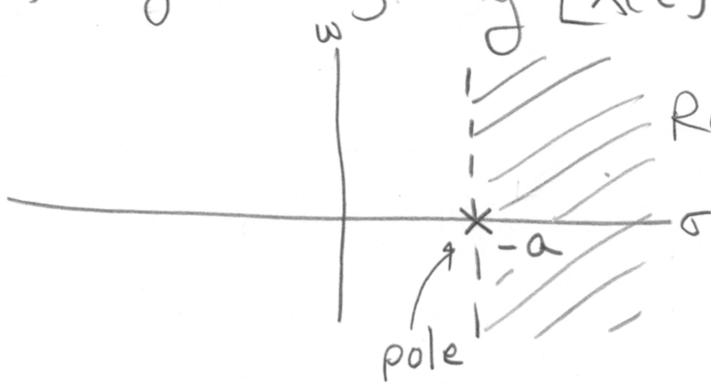
ROC contains ω-axis where $\sigma = 0$ so Fourier works, as seen before in RC circuit.

But what if $a < 0$? Now $x(t)$ expands!



unstable system (anthrax infection)

The math is the same, but ROC no longer contains ω-axis, so Fourier does not work. need to "stabilize" $x(t)$ by analyzing $[x(t)e^{-\sigma t}]$, $\sigma > -a$



in "one sided" Laplace Transform ($x(t) = 0$, $t < 0$)
ROC always to the right of all poles

using the prototype $e^{-at}u(t) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}$

here's an example:

$$x(t) = 3\underbrace{e^{-2t}}_{a=2}u(t) - 2\underbrace{e^{-t}}_{a=1}u(t)$$

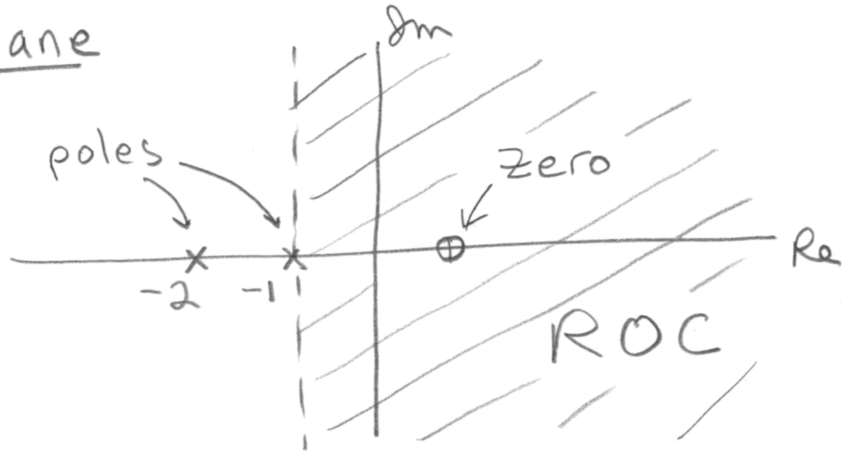
$$X(s) = \frac{3}{s+2} - \frac{2}{s+1}$$ Laplace is linear

$$= \frac{3s+3 - 2s-4}{(s+2)(s+1)} = \frac{(s-1)}{(s+2)(s+1)}$$

s-plane

$Re\{s\} = \sigma$

$Im\{s\} = \omega$



$Re\{s\} > -1$

Rational Laplace Transforms

$$X(s) = \frac{N(s)}{D(s)}$$

$N(s)$ ← roots of Numerator = zeros

$D(s)$ ← roots of Denominator = poles

ROC contains no poles, ever