Different Representations of a system

time domain, impulse response

$$\longrightarrow \boxed{h(t)} \longrightarrow$$

freq. domain, Fourier Transform (stable systems only)

$$\longrightarrow \boxed{H(\omega)} \longrightarrow$$

Laplace Transform (unstable systems OK)

$$\longrightarrow \boxed{H(s)} \longrightarrow$$

Diff. Eq.

eg, $y(t) + a\dfrac{dy(t)}{dt} = x(t)$ etc.

$H(s) \nearrow \dfrac{s+2}{s^2+5s+4} \longleftrightarrow$ Diff. Eq.

$H(s) \rightarrow \dfrac{(s+2)}{(s+1)(s+4)} \longleftrightarrow$ poles + zeros

$H(s) \searrow \dfrac{c_1}{(s+1)} + \dfrac{c_2}{(s+4)} \longleftrightarrow$ time domain (use partial fractions)

Recall the car swirving down
Mt. Washington.

needed:

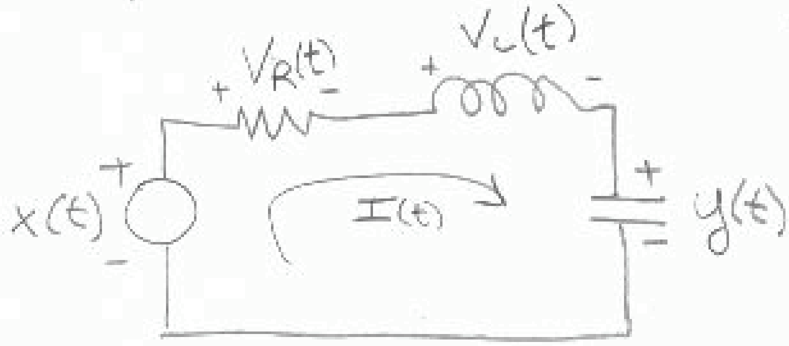
① source of energy (continual, independent of input signal) = gravity

② gain = steering wheel.

without these, the system will not be unstable, though it may exhibit "ringing"

for example ...

openheimer 9.24



$$I(t) = C\frac{dy(t)}{dt}$$

$$V_L(t) = L\frac{dI(t)}{dt} = LC\frac{d^2y(t)}{dt^2}$$

$$V_R(t) = RI(t) = RC\frac{dy(t)}{dt}$$

$$V_R(t) + V_L(t) + y(t) = x(t)$$

$$RC\frac{dy(t)}{dt} + LC\frac{d^2y(t)}{dt^2} + y(t) = x(t)$$

$$H(s) = \frac{1}{LCs^2 + RCs + 1}$$

$L, C, R > 0$

so

we will prove that the real part of the roots of denominator $D(s)$ must be negative, i.e. all poles in left half of s-plane $\Rightarrow$ stable.

passive electronics are <u>always</u> stable

You need gain to oscillate increasingly.

recall quadratic equation roots r_1 and r_2 = $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

"discrimmant" if <0 you have a complex conjugate pair of roots

$a = LC$

$b = RC$

$c = 1$

continued →

For the previous example assume discriminant < 0

Therefore, roots of $D(s)$ are

$a = LC$
$b = RC$
$c = 1$

$r_1 = \sigma + j\omega$

$r_2 = \sigma - j\omega = r_1^*$ where

$\underbrace{\qquad\qquad}_{\text{Re and Im parts}}$

$\sigma = -\dfrac{b}{2a} = -\dfrac{RC}{2LC} = -\dfrac{R}{2L} < 0$

$j\omega = \dfrac{\sqrt{b^2-4ac}}{2a} = \underbrace{\dfrac{\sqrt{(RC)^2-4LC}}{2LC}}_{\text{imaginary because}}$

imaginary because we assume

$(RC)^2 < 4LC$

$R^2C < 4L$

Thus it will "ring"....

$$H(s) = \frac{1}{(s-r_1)(s-r_2)} = \frac{C_1}{s-r_1} + \frac{C_2}{s-r_2}$$

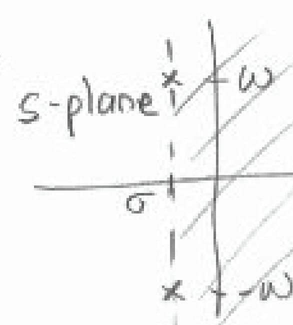
by partial fractions

$$C_1 = \Big[H(s)(s-r_1)\Big]_{s=r_1} = \frac{1}{r_1-r_1^*} = \frac{1}{2j\omega}$$

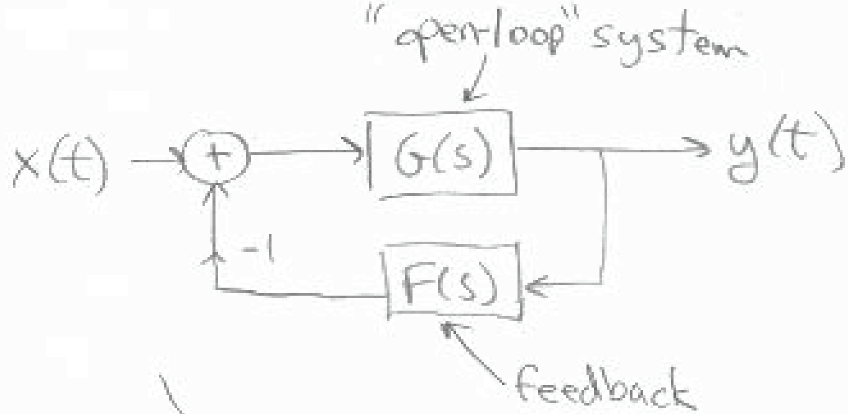
$$C_2 = \Big[H(s)(s-r_1^*)\Big]_{s=r_1^*} = \frac{1}{r_1^*-r_1} = -C_1 = -\frac{1}{2j\omega}$$

$$H(s) = \frac{1}{2j\omega}\Big[\underset{\uparrow\ \sigma+j\omega}{\frac{1}{s-r_1}} - \underset{\uparrow\ \sigma-j\omega}{\frac{1}{s-r_1^*}}\Big] \overset{\mathcal{L}}{\longleftrightarrow} h(t) = \frac{1}{\omega}e^{\sigma t}\left[\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right]u(t)$$

$$h(t) = \frac{1}{\omega}e^{\sigma t}\sin(\omega t)u(t)$$



stable since $\sigma < 0$



(see proof)

$$H(s) = \frac{G(s)}{1 + F(s)G(s)}$$

entire "closed-loop" system

unstable when $F(s)G(s) = -1$
because gain around the loop is 1

denominator becomes 0
(pole)

Example of car coming down
from Mt. Washington

energy from gravity
gain from steering wheel

<u>proof</u> $y(t) = [x(t) - y(t) * f(t)] * g(t)$

$$y(t) = x(t) * g(t) - y(t) * f(t) * g(t)$$

$$Y(s) = X(s)G(s) - Y(s)F(s)G(s)$$

$$Y(s) = X(s)\left[\frac{G(s)}{1 + F(s)G(s)}\right]$$

$$H(s) = \frac{Y(s)}{G(s)} = \frac{G(s)}{1 + F(s)G(s)}$$