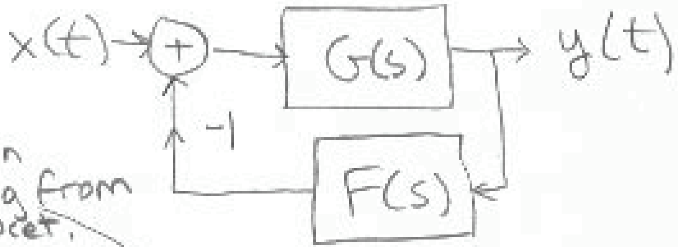
Now let's examine a system with an energy source and gain.

Controlling the temperature of the shower

$x(t)$ = rotational position of the valve

$y(t)$ = temperature



$$H(s) = \frac{G(s)}{1+F(s)G(s)}$$

τ is delay in water flowing from valve to faucet.

$$y(t) = a\,x(t-\tau) \qquad g(t) = a\,\delta(t-\tau) \overset{\mathcal{L}}{\longleftrightarrow} G(s) = a e^{-\tau s}$$

"open loop system" without F(s)

Now add feedback; turn knob by an amount proportional to perceived error in the temperature

$$f(t) = b\,\delta(t) \overset{\mathcal{L}}{\longleftrightarrow} F(s) = b \qquad \text{(remember the } -1\text{)}$$

$$H(s) = \frac{G(s)}{1+F(s)G(s)} = \frac{a e^{-\tau s}}{1 + ab e^{-\tau s}} \longleftarrow \text{denominator } D(s)$$

poles at roots of $D(s)$ where $ab e^{-\tau s} = -1$

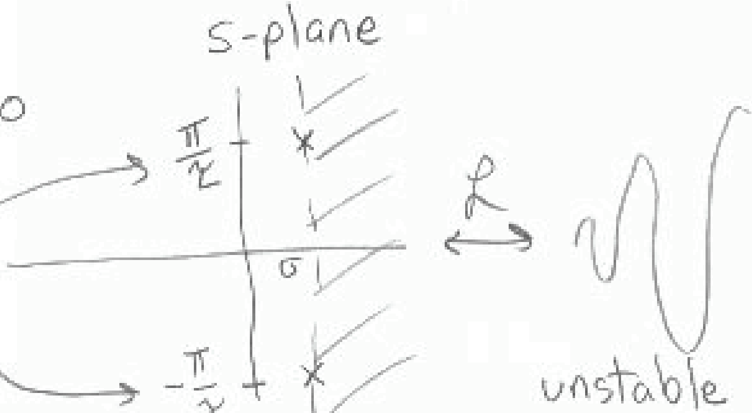
$$ab e^{-\tau\sigma} e^{-j\tau\omega} = -1$$

when

$$ab e^{-\tau\sigma} = 1 \Rightarrow \sigma = \frac{\ln(ab)}{\tau} > 0$$

and

$$\tau\omega = \pm\pi \Rightarrow \omega = \pm\frac{\pi}{\tau}$$



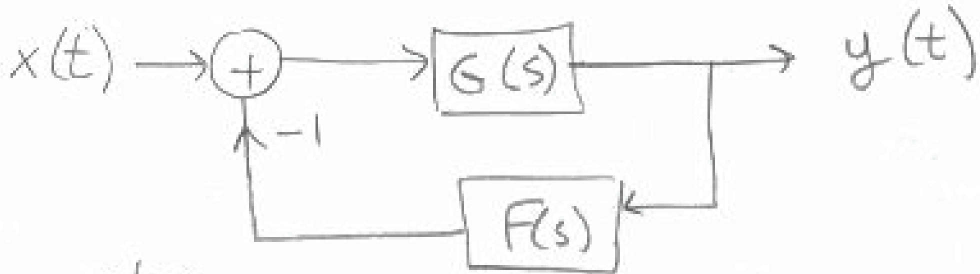
At this frequency you are doing exactly the wrong thing with the valve. 180° out of phase.

~~EXAMPLE 6.15 Bruce (PID)~~

~~Proportional, Integral, Derivative Feedback~~

~~Semi errata – Bruce~~
~~Fig 6.9, y(t) never mentioned in text, what are C(s) and A(s)?~~

PID – Proportional, Integral, Derivative (Feedback)



Here's an example:
assume $a>0$, $b>0$

open loop $G(s) = \frac{b}{s+a} \overset{\mathcal{L}}{\longleftrightarrow} b e^{-at} u(t)$ impulse response

1rst order

time constant $1/a$

① Proportional Feedback $F(s) = k$

$$H(s) = \frac{G(s)}{1+F(s)G(s)} = \frac{b}{(s+a)\left(1+k\left[\frac{b}{s+a}\right]\right)}$$

stable for $k \geq -\frac{a}{b}$

$$= \frac{b}{s+(a+bk)} \overset{\mathcal{L}}{\longleftrightarrow} b e^{-(a+bk)t} u(t)$$

1rst order

time constant $\frac{1}{a+bk}$ faster

② Derivative Feedback $F(s) = ks$

$$H(s) = \frac{G(s)}{1+F(s)G(s)} = \frac{b}{(s+a)\left(1+ks\left[\frac{b}{s+a}\right]\right)}$$

Stable for $k > -\frac{1}{b}$

$$= \frac{b}{s(1+kb)+a} = \frac{b/(1+kb)}{s + a/(1+kb)}$$

$$\updownarrow \mathcal{L}$$

$$\frac{b}{1+kb} e^{-\frac{a}{(1+kb)}t} u(t)$$

1rst order

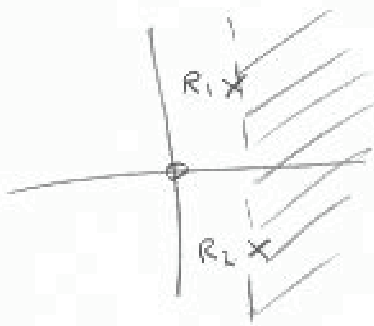
$\frac{1+kb}{a}$

time constant slower

③ Integral Feedback $F(s) = \frac{K}{s}$

$$H(s) = \frac{G(s)}{1+F(s)G(s)} = \frac{b}{(s+a)\left(1+\frac{K}{s}\frac{b}{(s+a)}\right)} = \frac{bs}{s^2+as+kb}$$

In this example, integral feedback adds a second pole, now a second order equation, can oscillate if roots of denominator are complex. If poles of $H(s)$ are not in left hand plane will be unstable



$$\frac{bs}{s^2+as+kb} = \frac{bs}{(s-R_1)(s-R_2)}$$

$R_1^* = R_2$ from quadratic equation

$$\cancel{\frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$ ~~discriminant~~

~~note different "a" "b" from above, as in~~ $\cancel{as^2+bs+c}$

Continuous in time | Discrete in time

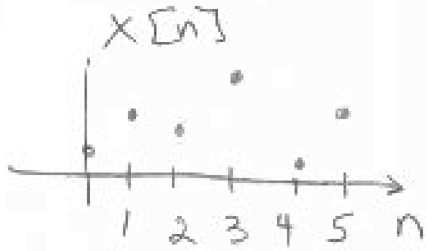
Fourier Series + Transform $\longrightarrow$ still Fourier

Laplace $\longrightarrow$ Z transform

In most cases, these signals are also discrete in value, i.e. quantized

$2\pi f_s$

sampling rate $= \omega_s$, sampling interval $t_s = \frac{2\pi}{\omega_s}$ sec.



$x[n] \cong x(nt_s) = x\left(\frac{2\pi n}{\omega_s}\right)$

eg $\omega_s = 2\pi 10 \frac{\text{radians}}{\text{sec}} = 10 \frac{\text{samples}}{\text{sec}}$

integer n has units of 0.1 sec

$t_s = 0.1$ sec

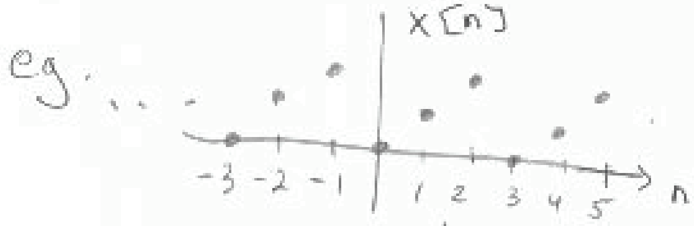
depends on how you sample

Now consider periodic discrete signals of the kind

$$x[n] = x[n+N]$$

i.e period is an integer number of samples

eg.



$N = 3$

Fundamental frequency of Fourier Series

$$\omega_0 = \frac{\omega_s}{N} = \frac{\omega_s}{3}$$

but the discrete notation "x[n]" ignores the actual sampling freq. ω_0, and so does the FS

So for now let us assume –

$$\tau_s = 1 \text{ sec}$$

$$\omega_s = 2\pi \ \frac{\text{radians}}{\text{sec}}$$

$$\omega_0 = \frac{2\pi}{N} \ \frac{\text{radians}}{\text{sec}}, \quad \left(f_0 = 1 \frac{\text{cycle}}{\text{sec}}\right)$$

How many independent terms would you expect in the Fourier Series of $x[n]$ when $N = 3$?

(The answer is 3)

The Fourier Series is <u>finite</u> for discrete signals. How can this be?
It is periodic (and discrete) in frequency.