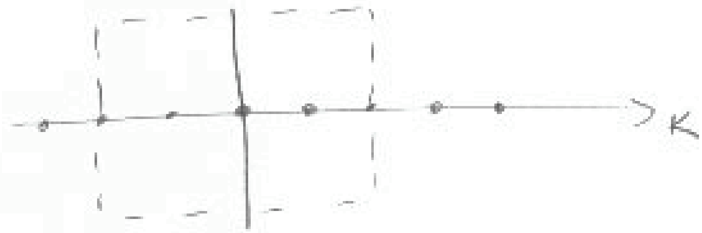
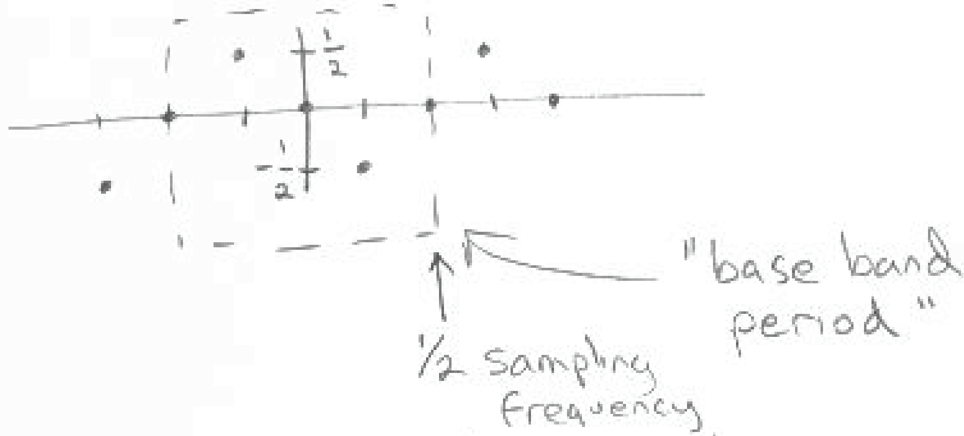
Example (continued)

$\mathrm{Re}\{a_k\}$



$\mathrm{Im}\{a_k\}$



how do 4 samples in time turn into 4 complex numbers (8 variables) in frequency?

since $x[n]$ is real

$a_k^* = a_{-k}$; only 4 independent variables -

Review of Discrete Fourier Transform

if $W = e^{j\frac{2\pi}{N}}$ then $W^N = 1$

Fourier Synthesis

$$x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\omega_0 n} \qquad \omega_0 = \frac{2\pi}{N}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ \vdots \\ x[N-1] \end{bmatrix} = \overbrace{\begin{bmatrix} W^0 & W^0 & W^0 & \cdots \\ W^0 & W^1 & W^2 & \\ W^0 & W^2 & W^4 & \\ \vdots & \vdots & \vdots & \\ \vdots & \vdots & \vdots & \end{bmatrix}}^{F} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_{N-1} \end{bmatrix}$$

1st harmonic

Fourier Analysis

$$a_k = \frac{1}{N} \sum_{n=\langle N\rangle} x[n] e^{-jk\omega_0 n}$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ \vdots \\ a_{N-1} \end{bmatrix} = \underbrace{\frac{1}{N} \begin{bmatrix} W^0 & W^0 & W^0 & \cdots \\ W^0 & W^{-1} & W^{-2} & \\ W^0 & W^{-2} & W^{-4} & \\ \vdots & \vdots & \vdots & \\ \vdots & \vdots & \vdots & \end{bmatrix}}_{F^{-1}} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ \vdots \\ \vdots \\ x[N-1] \end{bmatrix}$$

1st harmonic spinning backwards

Continuous	Discrete
Fourier Series	Discrete FS
	Fast Fourier "Transform" (FFT)
Fourier Transform	Discrete FT
Laplace Transform	Z - Transform

Discrete Fourier Transform

Synthesis

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(\omega)e^{j\omega n}\,d\omega$$

$+\pi \leftarrow \frac{\omega_s}{2}$

$-\pi \leftarrow -\frac{\omega_s}{2}$

other notations:

Hsu uses $X(\Omega)$

oppenheim uses $X(e^{j\omega})$

assume $\omega_s = 2\pi$

analysis

$$X(\omega) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$$

$$x[n] \overset{F}{\longleftrightarrow} X(\omega) = X(\omega + 2\pi)$$

DTFT are, by definition, periodic with a period of 2π.

Basically, in most cases we call the sampling frequency 1 sample/sec.

PROPERTIES OF DTFT

<u>Linearity</u> $a x_1[n] + b x_2[n] \xleftrightarrow{F} a X_1(\omega) + b X_2(\omega)$

<u>Delay</u> $x[n-n_0] \xleftrightarrow{F} e^{-j\omega n_0} X(\omega)$

<u>Differencing</u> $x[n] - x[n-1] \xleftrightarrow{F} (1-e^{-j\omega}) X(\omega)$

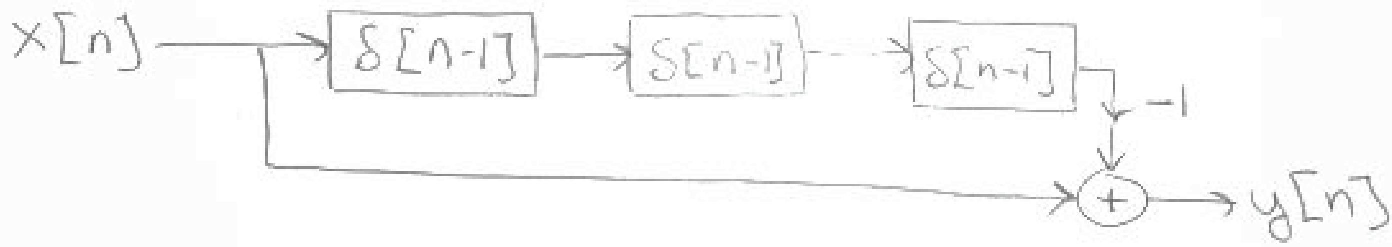
<u>Convolution</u> $y[n] = x[n] * h[n]$

$$Y(\omega) = X(\omega) H(\omega)$$

example

$$y[n] = x[n] - x[n-3]$$

sinusoids with a period of 3, or $\omega = \frac{2\pi}{3}$ will cancel themselves out.



Y and X terms are already separated (because it is a FIR system)

taking DTFT of both sides

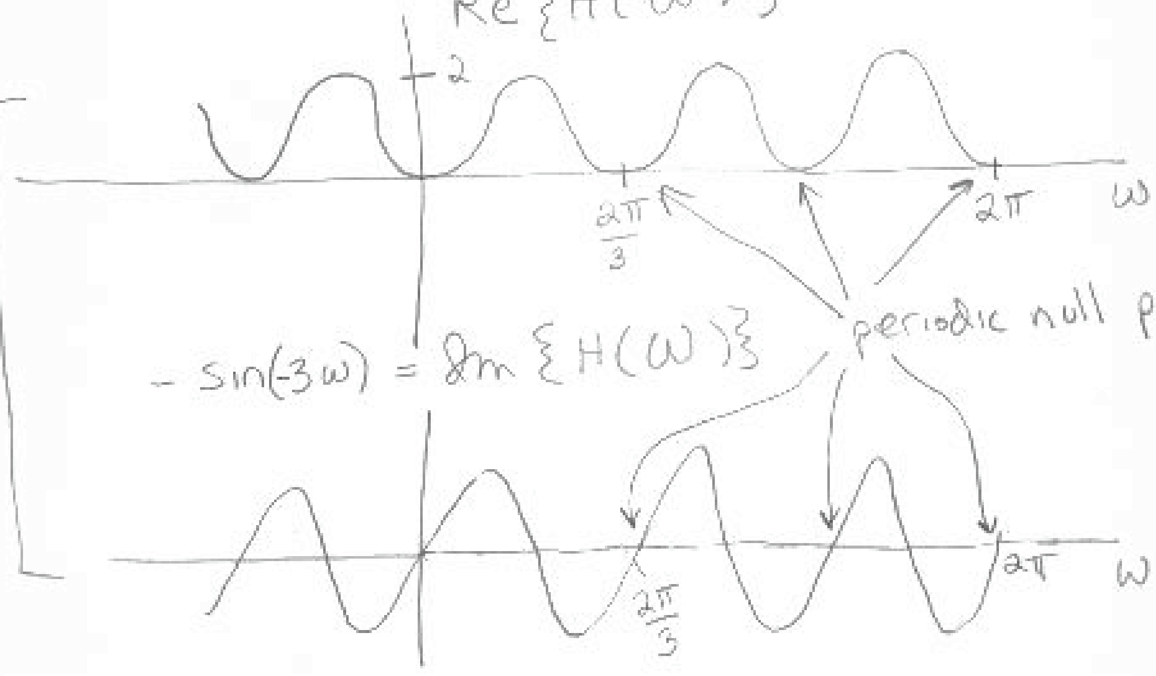
$$Y(\omega) = (1 - e^{-3j\omega}) X(\omega)$$

hint: we use Euler's Identity here: $e^{j\omega} = \cos\omega + j\sin\omega$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = 1 - e^{-3j\omega}$$ "comb filter"

$$Re\{H(\omega)\} = 1 - \cos(-3\omega)$$

$$-\sin(-3\omega) = Im\{H(\omega)\}$$



spectra still periodic with period of 2π