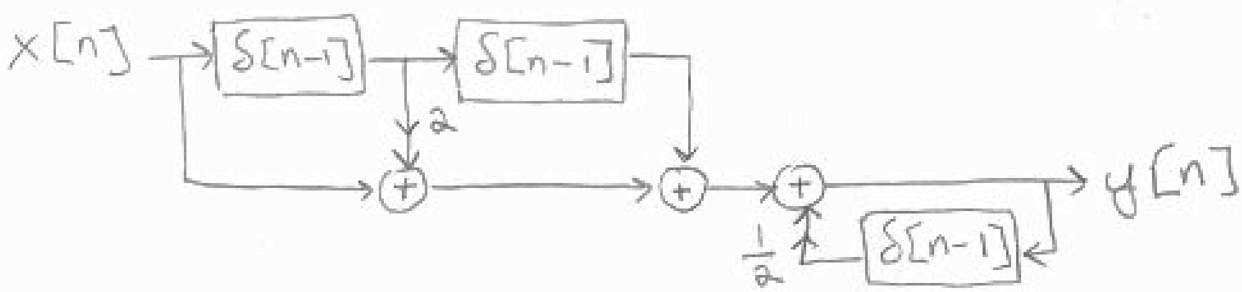
Example

Find the frequency response of a system that satisfies

$$y[n] = \frac{1}{2}y[n-1] + x[n] + 2x[n-1] + x[n-2]$$

these are easy to build. "Tapped delay line"



Set the y terms equal to the x terms

$$y[n] - \frac{1}{2}y[n-1] = x[n] + 2x[n-1] + x[n-2]$$

take the DTFT of both sides

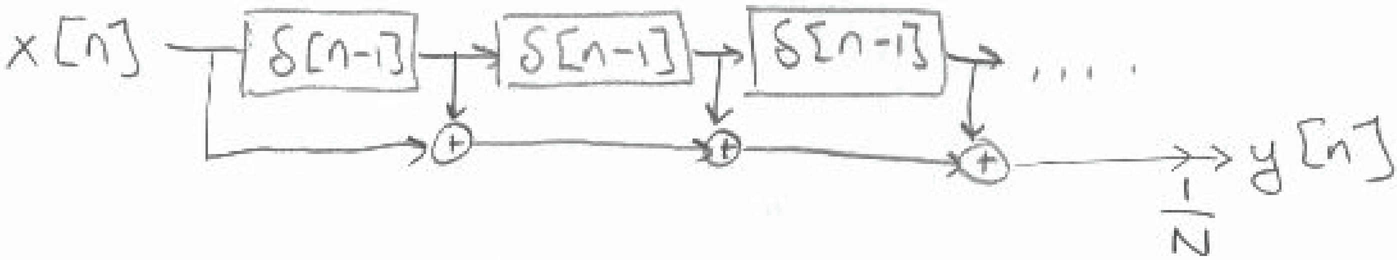
$$\left(1 - \frac{1}{2}e^{-j\omega}\right)Y(\omega) = \left(1 + 2e^{-j\omega} + e^{-2j\omega}\right)X(\omega)$$

since $H(\omega) = \dfrac{Y(\omega)}{X(\omega)}$

$$H(\omega) = \frac{1 + 2e^{-j\omega} + e^{-2j\omega}}{1 - \frac{1}{2}e^{-j\omega}}$$

Boxcar Filter – moving average

$$y[n] = \frac{1}{N}\left(x[n] + x[n-1] + x[n-2] \ldots x[n-(N-1)]\right)$$

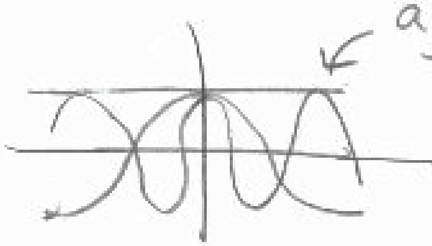


$$Y(\omega) = \frac{1}{N}X(\omega)\left(1 + e^{-j\omega} + e^{-2j\omega} \ldots\right)$$

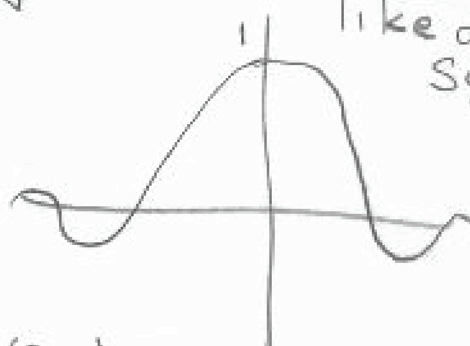
$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1 + e^{-j\omega} + e^{-2j\omega} \ldots}{N}$$

$$\mathrm{Re}\{H(\omega)\} = \frac{1 + \cos(\omega) + \cos(2\omega) \ldots}{N}$$

add these

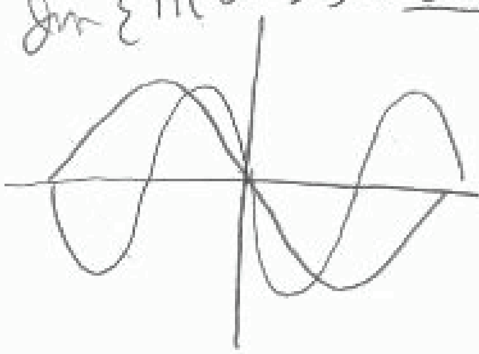


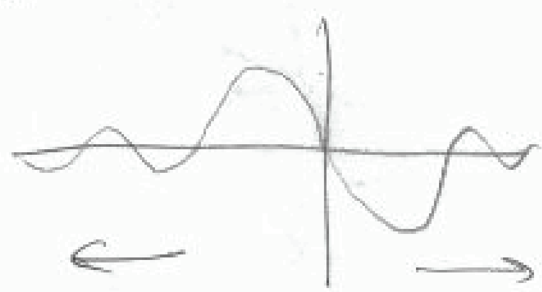
$\Rightarrow$



starts looking like a sync

$$\mathrm{Im}\{H(\omega)\} = \frac{-\sin(\omega) - \sin(2\omega) \ldots}{N}$$





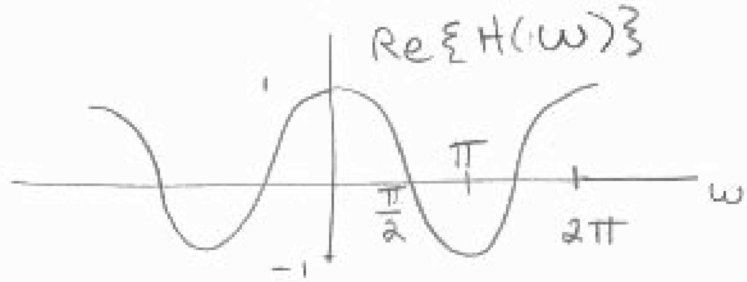
gets out of phase

Low Pass Filter

example

find the difference equation.

$$H(\omega) = \cos(\omega)$$

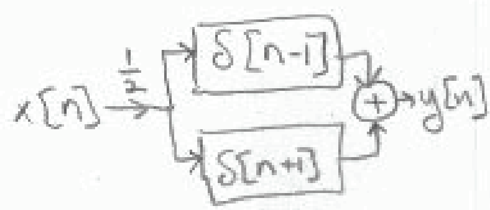


$$H(\omega) = \frac{e^{j\omega} + e^{-j\omega}}{2} = \frac{Y(\omega)}{X(\omega)}$$

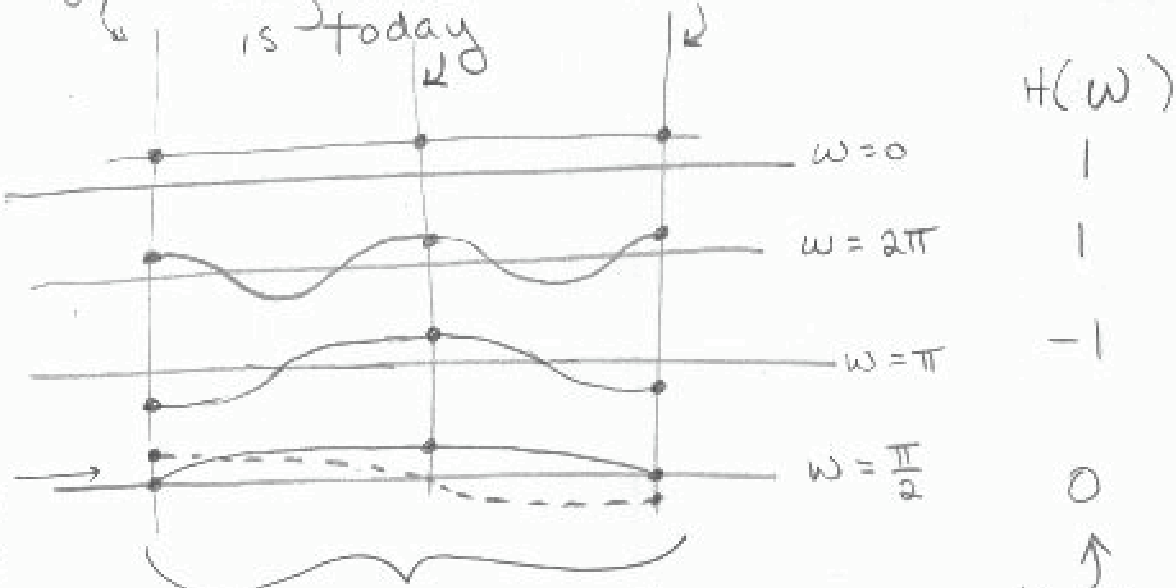
$$Y(\omega) = \left(\frac{e^{j\omega}}{2} + \frac{e^{-j\omega}}{2}\right) X(\omega)$$

take the inverse DTFT of both sides

$$y[n] = \frac{1}{2}\left(x[n+1] + x[n-1]\right)$$



average of yesterday and tomorrow is today



true for either phase

this signal will experience this gain

Now lets consider the actual sampling frequency, W_s, and not just call it 2π. We can predict whether a given analog signal will experience

"sampling artifact"

based on its component frequencies.

Non Periodic Discrete Signals – Discrete Four. Transform

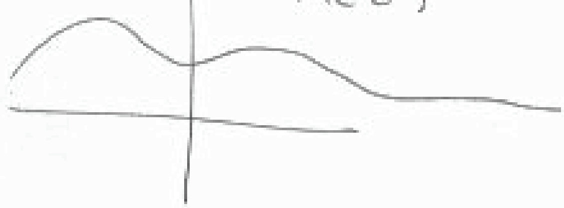
Impulse Train Sampling (oppenheim p516)

one way to represent $x[n]$ is

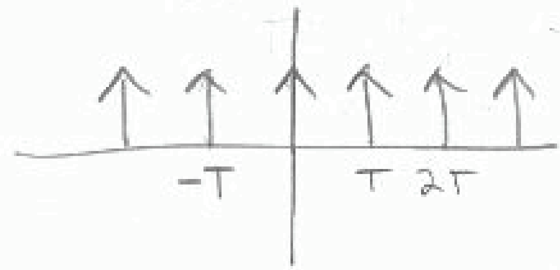
sampled signal $\rightarrow x_s(t) = x(t)s(t)$

$$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$ periodic impulse sampling function

$x(t)$



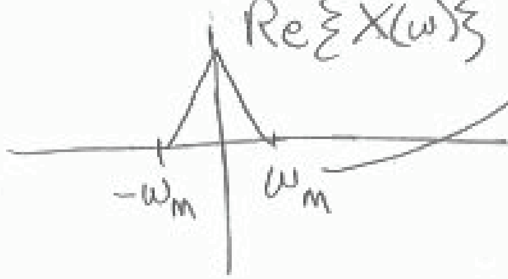
$s(t)$



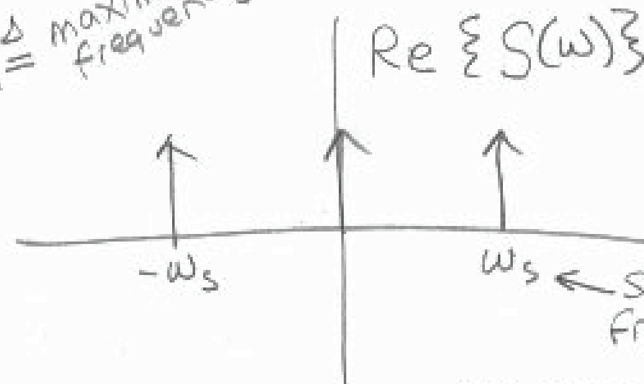
FT

$\omega_m \triangleq$ maximum frequency

$Re\{X(\omega)\}$

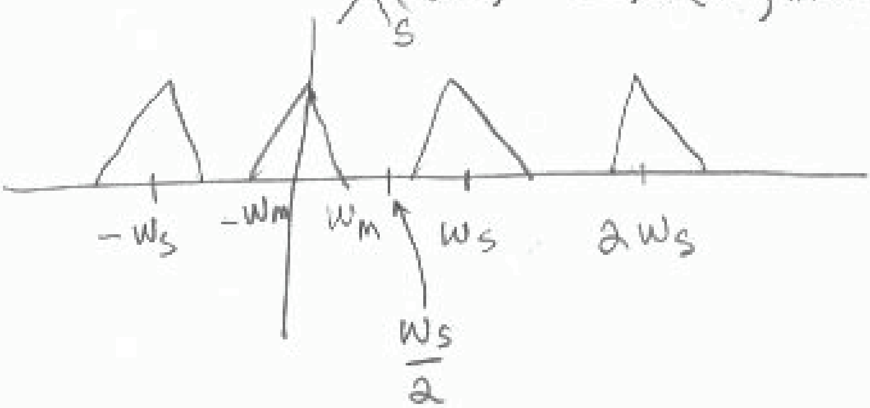


$Re\{S(\omega)\}$



$\omega_s \leftarrow$ sampling frequency

$X_s(\omega) = X(\omega) * S(\omega)$



NYQIST CRITERION

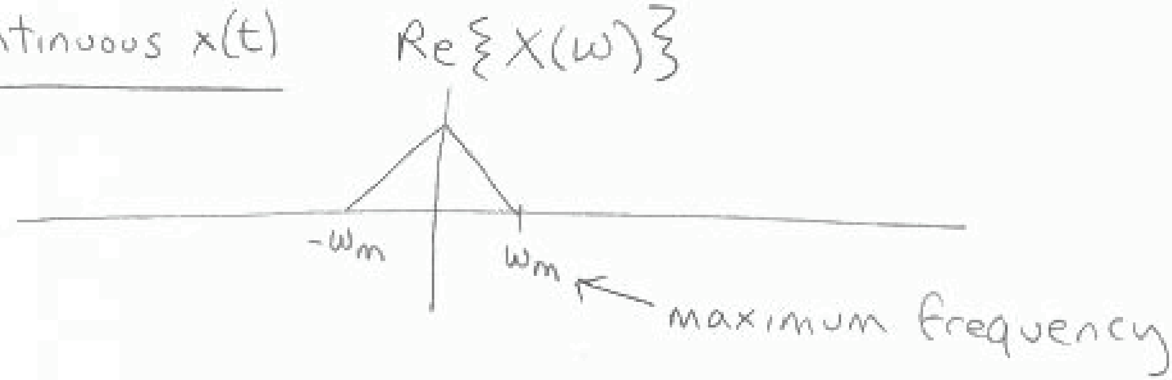
$$|\omega_m| < \frac{\omega_s}{2}$$

not "≤"

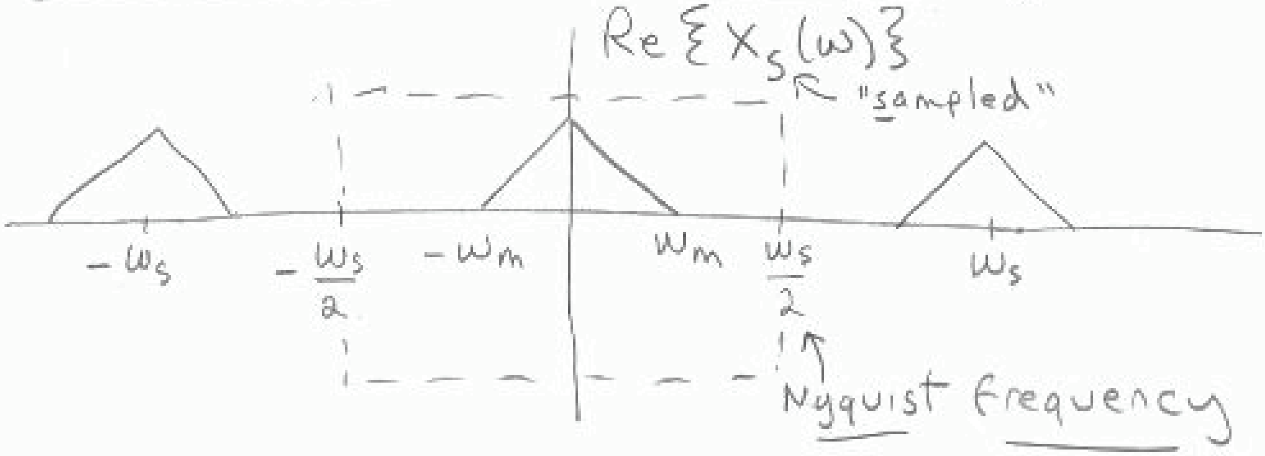
guarantee with analog low-pass filtering

aliasing of a continuous spectrum

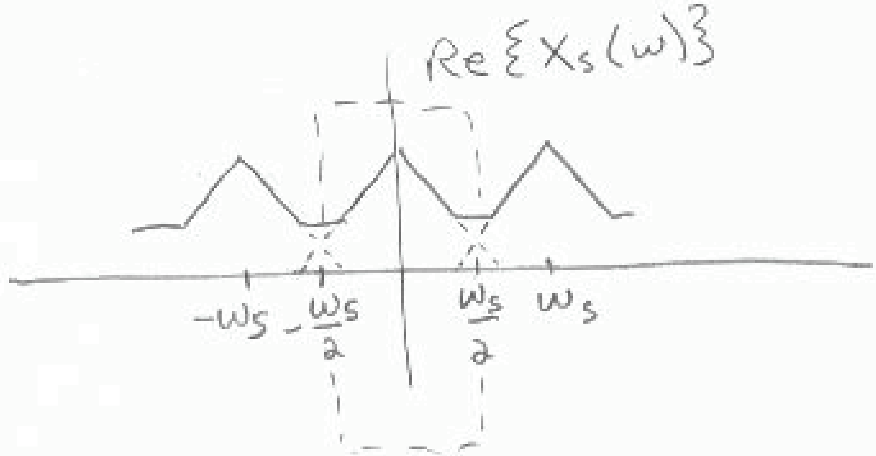
For continuous $x(t)$



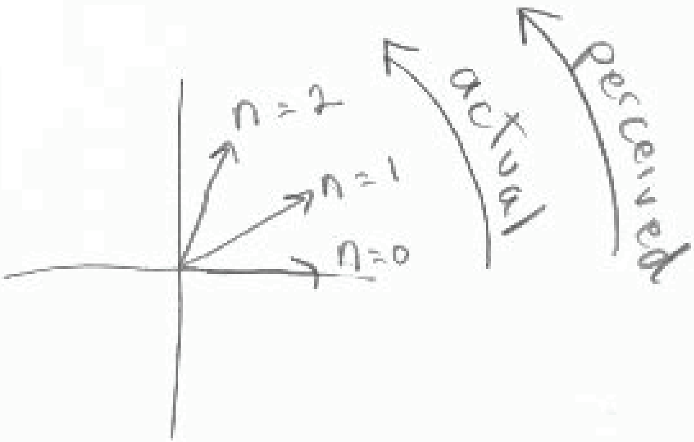
For $x(t)$ sampled at $\overbrace{\omega_s > 2\omega_m}^{\text{Nyquist criterion}}$, no aliasing



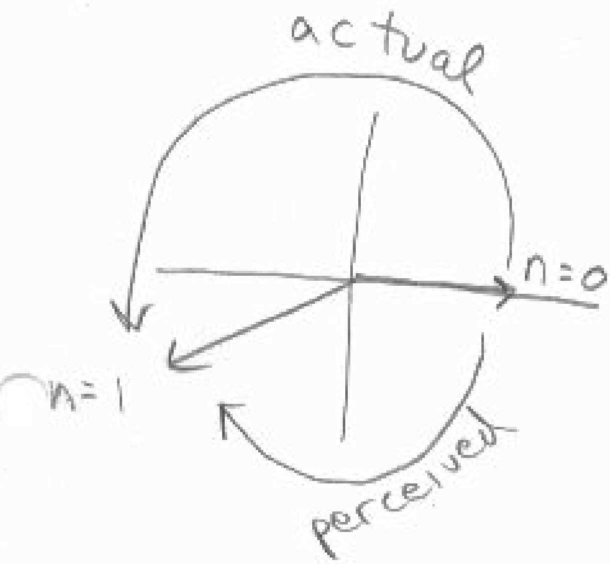
For $x(t)$ sampled at $\omega_s < 2\omega_m$, aliasing!



NYQUIST CRITERION

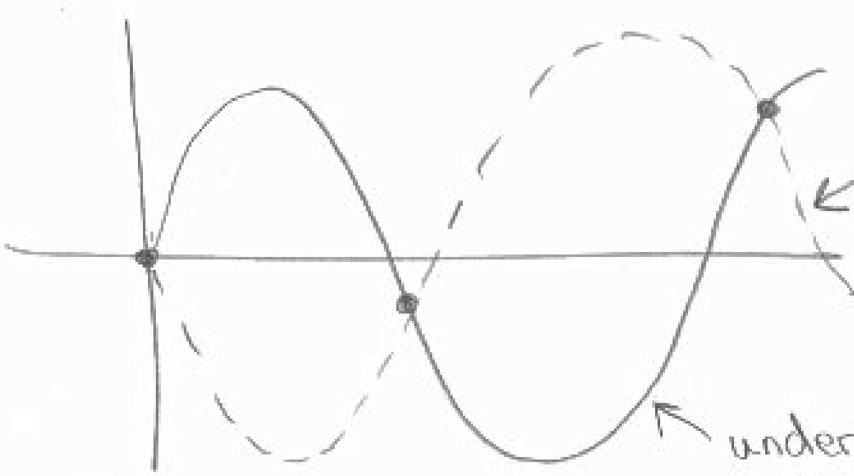


$> 2 \ \frac{\text{samples}}{\text{cycle}}$



$< 2 \ \frac{\text{samples}}{\text{cycle}}$

ALIASING



under sampled sin appears as lower frequency

(note that it is <u>negative</u>)